MARS GEODESY WITH NEIGE: SIMULATION OF THE MARTIAN ORIENTATION PARAMETERS ESTIMATION

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ABSTRACT

By analysing the radio Doppler shifts between an orbiter around Mars and a set of landers on Mars surface, and between this orbiter and the Earth, the NEtlander Ionosphere and Geodesy Experiment (NEIGE) will provide Mars orientation parameters, particularly polar motion, nutation and length-of-the-day (rotation rate) variations. Information about the martian core will be deduced from nutation estimations.

Due to the small amount of data that will be obtained (a few observations each week), the experiment objectives will only be achieved if an adapted strategy for the parameter adjustment is established. We have simulated the NEIGE data and the modeling of the rotational parameters. After 50 weeks, we obtain a precision in polar motion and rotation speed amplitude estimation at the level of a few mas.

Key words: NEIGE, nutation, rotation, Mars.

1. THE NETLANDER MISSION TO MARS

In 2008, four identical landers dedicated to network science will land on the red planet to study its internal structure and atmosphere. Nine experiments will be on board such as a seismometer, a magnetometer, meteorological sensors and a geodesy experiment. An orbiter will relay data from the station network to the Earth, and will also be used for the geodesy experiment. Several European countries and the USA participate in the NetLander mission. See Harri et al (1999) for more details.

2. NEIGE

The NEtlander Ionosphere and Geodesy Experiment, NEIGE, part of the NetLander mission, aims at measuring the Doppler shifts of two radio-links (1) between the NetLanders and the orbiter and (2) between the orbiter and the Earth (see Fig.1). Mars orientation parameters will be estimated from the analysis of the Doppler shifts, particularly the precession rate (long-term motion of the rotation axis around an axis perpendicular to the ecliptic), the nutations (periodic motions of the rotation axis in space, see Fig.2), the polar motion (motion of the rotation axis in a frame tied to the planet) and the rotation rate variations (length-of-day).

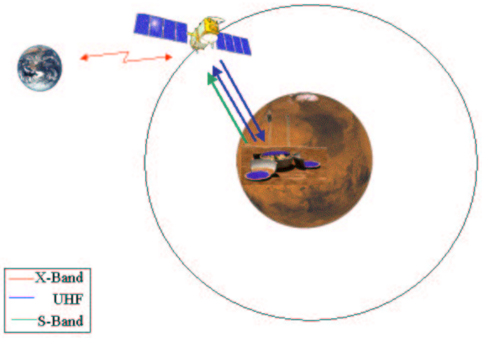

Figure 1. Principle of the NEIGE Experiment.

As nutations are influenced by the interior structure of Mars, information about the martian core, such as the core dimension, the density or the physical state, will be deduced from nutation estimations (Dehant et al, 2000, Van Hoolst et al, 2000a and 2000b, Van Hoolst et al, this issue). Seasonal exchange of CO_2 between the atmosphere and the ice caps will also be investigated from the annual and semi-annual variations of the rotation rate (Defraigne et al, 2000).

In addition to these geodesy objectives, ionosphere properties like the total electronic content will be deduced from the perturbation on the radio-signals (Morel et al, 2002). See also Barriot et al (2001) or

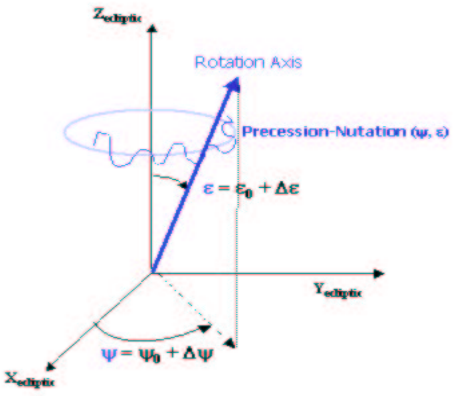

Figure 2. Definition of the nutation angles $\Delta\psi$ and $\Delta\epsilon$.

the web site cited at the end of references for more information about NEIGE.

3. MARTIAN ORIENTATION PARAMETERS

In this study, we want to test the quality of the adjustment of the Martian rotation parameters. Due to the small number of measurements that will be available in 2008-09 (1 pass/lander/week), it is appropriate to use only a limited number of parameters to adjust.

Therefore, we evaluate polar motion (PM) not as a temporal series, but as a sum of annual, semi-annual and Chandler frequencies for each component (X and Y polar motion). This means that only 12 parameters (two parameters for each frequency) must be determined instead of about 100 coefficients (one each week during one terrestrial year). The Martian Chandler period is taken equal to 200 days (Van Hoolst et al, 2000a). For the rotation rate variations, the 50 parameters of the temporal series have been reduced to four parameters for the annual and semi-annual frequencies.

Parameter	#	Frequency	Name
Polar Motion (X)	6	Annual	PX1
		Semi-annual	PX2
		Chandler Wobble	PXC
Polar Motion (Y)	6	Annual	PY1
		Semi-annual	PY2
		Chandler Wobble	PYC
Rotation rate	4	Annual	UT1
		Semi-annual	UT2
Nutations	2	F and σ_0	NUT

Table 1: The 18 rotation parameters that will be adjusted

We didn't investigate precession modeling in this study, as the precession period is about 175 000 years, which is large compared to the one year period of data acquisition.

Nutation can also be modeled by trigonometric series modeling instead of temporal series (100 parameters if $\Delta\psi$ and $\Delta\epsilon$ are estimated each week during one terrestrial year). The main frequencies to be investigated are the 6 nutations induced by the Sun. Nutations induced by Phobos and Deimos are very small. The amplitude and phases of the rigid nutations can be computed from astronomical theory with a high precision, see Roosbeek (2000) for example. To each rigid nutation, a non-rigid part must be added. In particular, the response of Mars to the forcing of the Sun could be influenced by the resonance of a possible fluid core. This can be mathematically be accounted for by convolving (a product in the frequency domain) a transfer function with the rigid nutation series (Folkner et al, 1997, Sasao et al, 1980):

$$r'_m = r_m(1 + F\frac{\sigma_m}{\sigma_m + \sigma_0}) \quad (1)$$

$$p'_m = p_m(1 + F\frac{\sigma_m}{\sigma_m - \sigma_0}) \quad (2)$$

where p_m and r_m are the rigid prograde and retrograde amplitudes of the nutations for the angular frequency σ_m(annual, semi-annual,...), and p'_m and r'_m are the prograde and retrograde amplitudes of the nutations after applying the transfer function, F and σ_0 are the two unknowns, they are a function of the core physical state, size and flattening. F is zero for a solid core and about 0.02 for a liquid core; it mainly depends on the polar moment of inertia of the fluid core. σ_0 is the frequency of the Free Core Nutation (FCN) which is a rotational normal mode connected with the fluid core. Differences between rigid and non-rigid nutations are shown in Fig.3.

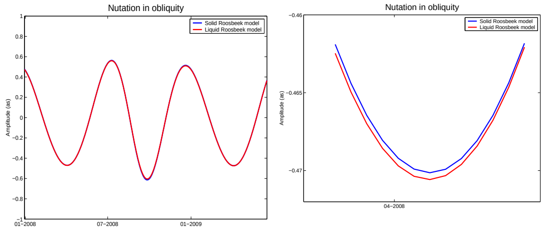

Figure 3. Nutation in obliquity ($\Delta\epsilon$) as a function of time. Nutation with a solid core (Blue) versus nutation with a liquid core (Red). The right graph zooms on the first minimum of the nutation in obliquity (left graph).

$F = 0$	No liquid core
$F = 0.02$	Liquid core with a 1468 km radius
$F = 0.2$	Large liquid core with a 1700 km radius
$\sigma_0 = -2\pi/240$	Estimated FCN rate with a 1468 km core radius (rad/day)

Table 2: Numerical values for nutation convolution parameters F and σ_0.

Together with the 12 parameters for PM and the four parameters for the rotation rate, a total of 18 parameters must then be determined.

4. SIMULATIONS

Only a small number of measurements will be made because of power limitations on the landers. In order to evaluate the number of weeks of observation needed to correctly model the interior structure of Mars, we determined the evolution of the rotational parameters precision and of their associated variances with observation time, by using both analytical and numerical simulations.

4.1. Numerical simulations

Synthetic Doppler shifts between the landers and the orbiter, and between the Earth and the orbiter have been simulated over 100 arcs of one week each. The orbiter's initial position and velocity have been adjusted for each arc, while the 18 orientation parameters defined in § 3 have been globally adjusted. The orbitography software used was GINS ("Géodésie par Intégrations Numériques Simultanées" developed in the GRGS Toulouse), which makes estimation of geophysical and orbit parameters through a least squares process. Each frequency is modeled by a cosine and sine amplitude.

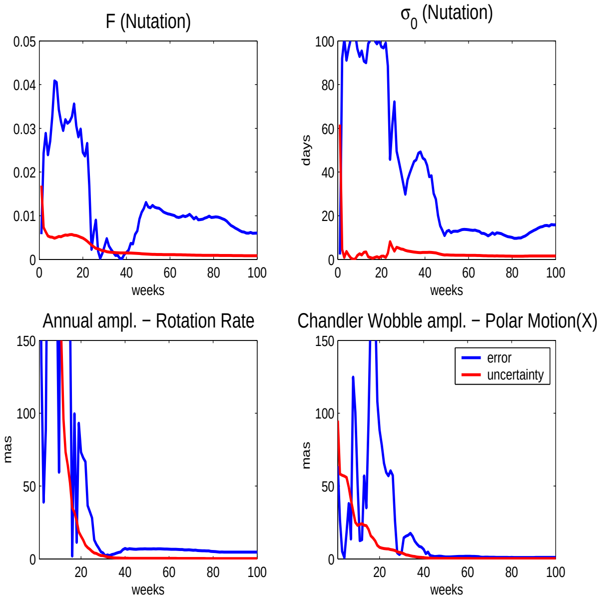

Figure 4. Test of the numerical adjustment of four parameters among 18: true errors (in blue) and a posteriori standard deviation (in red), as a function of the observation time.

In Fig.4, true errors and a posteriori standard deviations are plotted as a function of the observational time. The "true error" is defined as the difference between the value of the parameter used for the data simulation and the value coming out with the least square process. "True errors" usually decrease with the observation time. The behaviour during the first observation weeks is chaotic, probably because the amount of available data is too small. For F and σ_0, we need to iterate the least squares process in order to obtain smaller errors. This means that the adjusted value coming out with the process is used as an a priori model for the next iteration of the adjustement to the simulated data. After a second iteration, the true error is very close to the uncertainty. The configuration used is a network of four landers on Mars (three landers around Tharsis, one in Hellas impact bassin), and a 500 km circular heliosynchronous orbit. Observations are performed during one pass/lander/week.

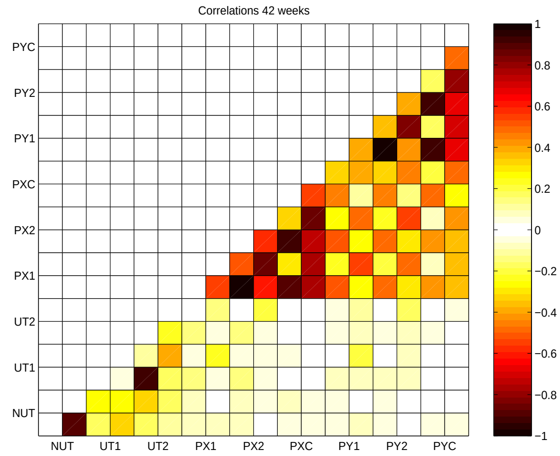

Figure 5. Numerical correlation between the 18 model parameters after 42 weeks for a network of four landers, and a 500 km circular heliosynchronous orbit (93 ° inclination). See Table 1 for label axis explanation.

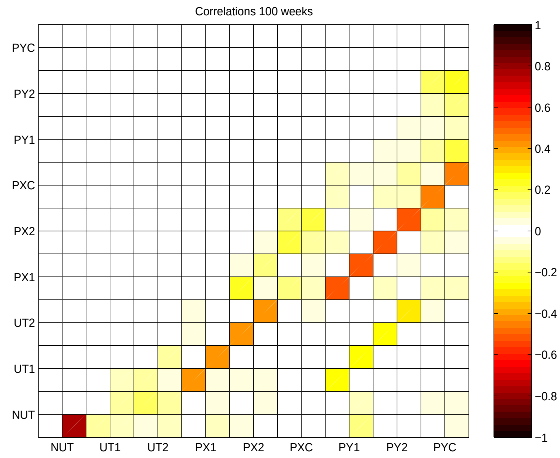

Figure 6. Numerical correlation between the 18 model parameters after 100 weeks for a network of four landers, and a 500 km circular heliosynchronous orbit.

Figs.5 and 6 show the evolution of the correlations between the 18 model parameters w.r.t. time. The correlations between parameters decrease with time, they are still large after 30 weeks, but largely decrease after 100 weeks (around one martian year). From Fig.6, we see that rotation rate and PM components have larger correlations if they correspond to the same frequency.

4.2. Analytical simulations

In this section, the Doppler observable is expressed analytically as a function of configuration parameters (the lander and orbiter positions and velocities) and the rotation parameters between the inertial and the planetary frames. The Doppler expression has been derived with respect to the 18 rotation parameters. A variance-covariance study has then been performed for a particular orbit by evaluating the derivatives once per week per lander. Each frequency is modeled by a phase and an amplitude.

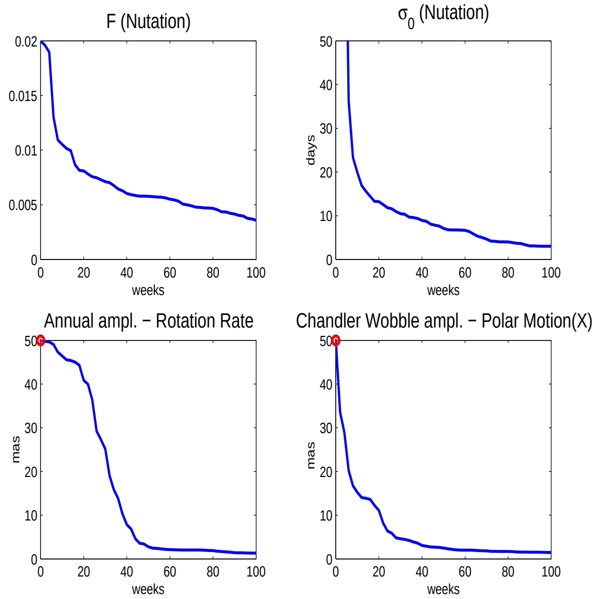

Figure 7. Standard deviation of four parameters among 18 as a function of the observational time (obtained by a variance-covariance analysis). The red dots are the a priori standard deviations.

We plotted the a posteriori standard deviations as a function of the observational time in Fig.7. One can see that they decrease very fast to small values w.r.t. time. Their behaviour is very similar to the true error after the second iteration and to the uncertainties found in the numerical simulations. We used the same configuration as for the numerical study.

The time evolution of the correlation between 18 parameters has also been investigated (Figs.8 and 9). One can see that they also decrease with the observation time.

For comparison, correlations between 18 parameters after 100 weeks by considering only one lander have been computed by considering the same orbit, see Fig.10. As expected, the correlations between the

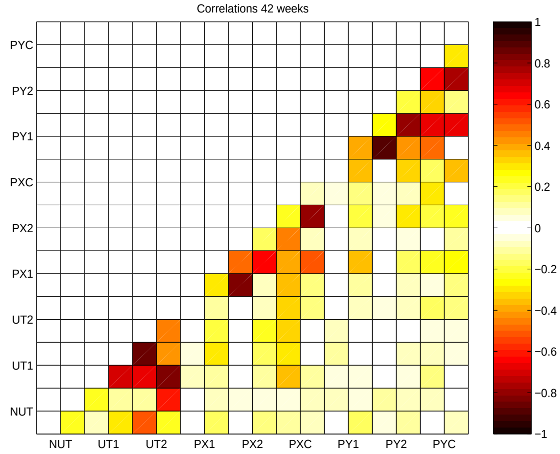

Figure 8. Analytical correlation between the 18 model parameters after 42 weeks for a network of four landers, and a 500 km circular heliosynchronous orbit (93 ° inclination).

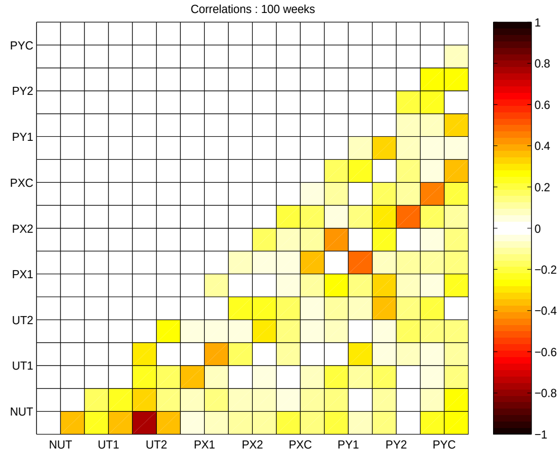

Figure 9. Analytical correlation between the 18 model parameters after 100 weeks for a network of four landers, and a 500 km circular heliosynchronous orbit.

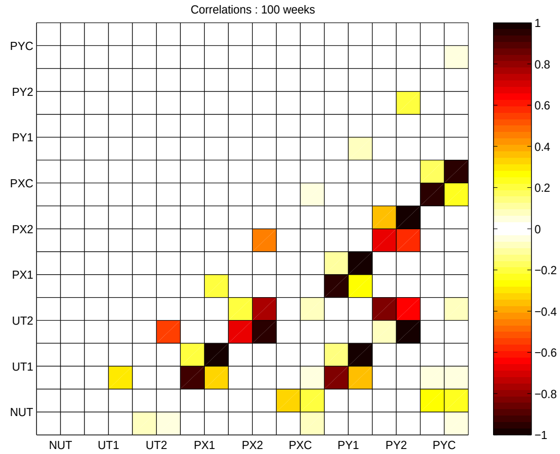

Figure 10. Correlation between the 18 model parameters after 100 weeks for only one lander, same orbit.

parameters are large because of the missing lander-orbiter geometries.

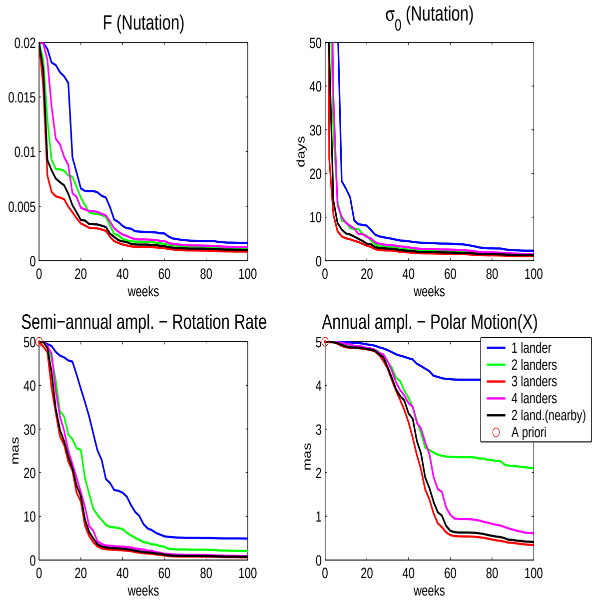

Figure 11. Standard deviation of four rotation parameters as a function of the observational time (obtained by a variance-covariance analysis), for different numbers of lander.

In Fig.11, we compare the parameter variances as a function of the number of landers included in the network. We see that some parameters like polar motion components can not be determined with only one or two nearby landers.

5. CONCLUSIONS

Our simulations show that the best strategy in order to have small uncertainties on the rotation angles for the NEIGE experiment is to minimize the number of parameters to solve, given the small amount of data. The different geophysical signals have been adjusted through a sum of frequencies for polar motion and length-of-day. Two parameters (F and σ_0) have been used to evaluate the effect of the core on the nutations.

After 50 weeks, the correlations between the parameters are small, and we obtain a precision in polar motion and rotation rate amplitude estimations of a few mas. This will permit in particular to model accurately the CO_2 sublimation-condensation processes. Furthermore, F and σ_0 will be known with sufficient accuracy to be used for core properties determination if four landers are used.

With only one lander, the correlations between parameters are too large and the precision obtained is not sufficient to meet the NEIGE objectives. Three landers on Mars' surface give approximately the same results as four landers. If only two geometrically well spread landers (not on the same latitude or longitude, distance larger than 5 000 km) are used, the same precision can be obtained by monitoring the rotation parameters over a longer period of time (some weeks more).

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