On the distance of Magellanic Clouds: first overtone Cepheids

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ABSTRACT

We present a detailed comparison between predicted and empirical $PL_{I,K}$ relations and Wesenheit function for Galactic and Magellanic Clouds (MCs) First Overtone (FO) Cepheids. We find that zero-points predicted by Galactic Cepheid models based on a noncanonical (mild overshooting) Mass-Luminosity (ML) relation are in very good agreement with empirical zero-points based on HIPPARCOS parallaxes, while those based on canonical (no overshooting) ML relation are $\approx 0.2-0.3$ mag brighter. We also find that predicted and empirical PL_K relation and Wesenheit function give, according to optical (V, I OGLE) and near-infrared (NIR, K, 2MASS) data, mean distances to the MCs that agree at the 2% level. Individual distances to the Large and the Small Cloud are: 18.53±0.08-19.04±0.11 (theory) and 18.48±0.13-19.01±0.13 (empirical). Moreover, predicted and empirical FO relations do not present, within the errors, a metallicity dependence. Finaly, we find that the upper limit in the FO period distribution is a robust observable to constrain the accuracy of pulsation models. Current models agree within 0.1 in $\log P$ with the observed FO upper limits.

Subject headings: Cepheids – Magellanic Clouds – stars: distances – stars: evolution – stars: oscillations

1. Introduction

The massive photometric databases collected by the micro-lensing experiments (MACHO, EROS, OGLE) substantially increased the number of known variable stars in the Galaxy and in the MCs. They also provided the unique opportunity to improve the sampling along the light curves of fundamental (F) and overtone pulsators. In particular, the new multi-band data on classical Cepheids have had a substantial impact not only on the pulsation properties of these variables but also on the estimate of MC distances (Udalski 1998; Groenewegen & Oudmaijer 2000, GO00;

Groenewegen 2000, G00; Bono, Caputo, & Marconi 2001, BCM01). Even though F Cepheids are the most popular standard candles to estimate distances, several theoretical and empirical investigations have been recently focused on Cepheids pulsating in the first (G00; Feuchtinger, Buchler & Kolláth 2000, FBK00; Baraffe & Alibert 2001, BA01) or in the second overtone (BCM01). The main advantage in using overtone pulsators to estimate distances is that the width in temperature of their instability regions is significantly smaller than for the fundamental one. As a matter of fact, current predictions suggest that at $\log P = 0.3$ the width of FO instability strip is 400 K, while for F variables it is 900 K at $\log P = 1$. Therefore, distances based on FO Period-Luminosity (PL) relations are marginally affected by intrinsic spread when compared with F ones. However, we still lack a comprehensive analysis of the uncertainties affecting distance estimates based on optical and NIR PL relations and on the Wesenheit function. A similar approach was already adopted by G00 but in the comparison between theory and observations he was forced to fundamentalize the period, since theoretical predictions for FO in MCs were not available. This gap was filled by BA01 who estimated PL, PLC relations, and Wesenheit function on the basis of linear, convective models. Although FO variables present several undisputable advantages, complete and accurate samples are only available for MC Cepheids. Moreover, the detection of FOs in external galaxies is more difficult than for F Cepheids, since they are fainter and the luminosity amplitudes are smaller.

The main aim of this Letter is to give a detailed analysis of pros and cons of the PL relations currently adopted to estimate the distances, according to full amplitude, nonlinear, convective models of Galactic and MC Cepheids. We are also interested in testing whether current models account for the observed upper limit in the FO period distribution.

2. Comparison between theory and observations

To investigate the topology of the instability strip of Galactic and MC Cepheids, pulsation models were constructed by adopting three different chemical compositions, namely Y=0.25, Z=0.004 (Small Magellanic Cloud, SMC), Y=0.25, Z=0.008 (Large Magellanic Cloud, LMC), and Y=0.28, Z=0.02 (Galaxy). To assess whether the FO instability strip presents a change in the slope when moving from long- to short-periods we adopted several stellar masses. For each mass value and chemical composition, the luminosities were fixed according to the same Mass-Luminosity (ML) relation adopted in previous investigations (Bono, Marconi & Stellingwerf 1999, BMS99) which is consistent, within the errors, with the ML relation derived by Bono et al. (2000). To mimic the luminosities predicted by evolutionary models that account for mild convective core overshooting during H-burning phases (noncanonical) the luminosities predicted by canonical models were increased by 0.25 dex (Chiosi, Wood & Capitanio 1993). Recent evolutionary predictions by Girardi et al. (2000) support such a scaling. This choice was driven by the fact that Cepheid models constructed using the same theoretical framework and a noncanonical ML relation account for the luminosity variation over the pulsation cycle of two LMC bump Cepheids (Bono, Castellani, & Marconi 2002). The same outcome does not apply to the models based on a canonical ML relation.

The models at solar chemical composition are the ones by Bono et al. (2001). As for the other chemical compositions, we constructed 5 new FO sequences with stellar masses ranging from $M/M_{\odot} = 3.0$, to 5.6 and stellar luminosities ranging from $\log L/L_{\odot} = 2.58$, to 3.60 to constrain the instability strip in the short-period range. The input physics and the physical assumptions adopted to construct the models are discussed in BMS99. For each model the stability analysis was only performed for the first two modes. The physical structure of the static envelope was forced out of equilibrium by perturbing the radial eigenfunction with a velocity of 5 km s^{-1}. Each case was integrated in time till the similarity over consecutive cycles was of the order of 10^{-4} or smaller. Close to the instability edges the temperature step is of 100 K. Bolometric light curves were transformed into the observational plane using the atmosphere models by Castelli et al. (1997). The linear regressions to estimate PL relations and the reddening-free Wesenheit function were performed at fixed chemical composition. We adopted this approach because previous relations are affected by different systematic uncertainties. The split of the FO sample into three sub-samples (Galactic, MCs) reduces the intrinsic spread of empirical relations. Therefore the comparison between theory and observations should allow us to single out the dependence on metallicity, and/or reddening.

In Table 1 theoretical slopes and zero-points of various PL-relations are listed for canonical and noncanonical FO pulsation models at solar metallicity, and compared to empirical zero-points based on HIPPARCOS data. Note that observed zero-points have been evaluated using the theoretical slopes. It is recalled that the accuracy of the HIPPARCOS data for Cepheids was not sufficient to determine both zero-point and slope of the Galactic PL-relation. However, for an adopted slope, it is possible to derive in a bias-free manner the zero-point using the method of "reduced parallax" (Feast & Catchpole 1997; GO00). Here, for the first time, HIPPARCOS data will be used to derive the zero-point of FO PL-relations adopting the same procedure. Note however that the periods need still to be "fundamentalized" to apply the appropriate reddening corrections (see GO00 for details). The results listed in Table 1 show that predicted noncanonical FO zero-points are in good agreement with empirical ones both in optical and NIR bands. On the other hand, FO zero-points predicted by canonical models are 0.2-0.3 mag systematically brighter than empirical ones. A similar test was also performed using F pulsators and we found that the difference between predicted noncanonical and empirical zero-points ranges from 0.08 (W), 0.2 (PL_K), to 0.6 (PL_V). It is not clear whether this discrepancy is due to problems in the reddening correction or to a difference in the period cut-off. However, we did not find a systematic difference between canonical and noncanonical zero-points. This suggests that FO variables are more robust to constrain theoretical models, however more extended calculations are required to constrain the difference. This finding strengthens the result obtained by Bono et al. (2002) concerning the accuracy of nonlinear, convective, bump Cepheid models. They found that theoretical models constructed by adopting a canonical ML relation do not account for the luminosity change over the pulsation cycle of two LMC bump Cepheids.

To provide independent MC distance evaluations we performed a linear regression over theoretical models for Z=0.008, 0.004. Table 2 gives zero-points, slopes, and distances obtained using theoretical relations. The MC FO sample is based on OGLE (V and I) and on 2MASS (K) data, reddening corrections (see G00 for details) come from OGLE estimates (Udalski et al. 1999; Bersier 2000). The empirical zero-point is derived by fixing the slope of the PL-relation to the theoretical one. Figs. 1 and 2 show that the slopes found by fitting the data alone differ slightly from the theoretical slopes. However, this is allowed for by the larger error-bars in the empirical zero-point. Note that lower-limits in period are enforced to avoid problems with a possible nonlinear PL-relation at short periods in the SMC and the effect of Malmquist bias (see G00). We applied 3σ clipping.

The results listed in Table 2 clearly show that the true LMC distances based on different distance indicators agree quite well and range from 18.42 ± 0.17 (PL_I) to 18.54 ± 0.09 (W). The small discrepancy between the distance based on PL_I relation and the ones based on PL_K and W function could be due to a mild overestimate of interstellar reddening. In fact, both PL_K and W function are only marginally affected by reddening corrections. The same outcome applies to the SMC and indeed the evaluated distance ranges from 18.98 ± 0.17 (PL_I) to 19.06 ± 0.13 (W). However, it is noteworthy that the relative distance between the clouds attains the same value, namely 0.51 (PL_K), 0.54 (W), and 0.56 (PL_I) and agree with relative distances provided by Udalski et al. (1999) and by G00. Moreover, and even more importantly, these distances are in good agreement with empirical distances based on observed MC slopes and zero-points. Note that the "equivalent Galactic zero-points" have been estimated using HIPPARCOS FO data and the empirical MC slopes. The data listed in Table 3 show that empirical solutions provide LMC distances that range from 18.28 ± 0.16 (PL_I) to 18.45 ± 0.35 (PL_K) and 18.48 ± 0.14 (W). At the same time, SMC distances range from 18.91 ± 0.17 (PL_I) to 19.02 ± 0.14 (W). Note that the difference in the slope between LMC and SMC FOs is due to the adopted period cutoff. The distance moduli obtained in G00 for F pulsators using the same method are typically 0.1 larger. Once again, absolute distances based on the PL_I relation attain smaller values when compared with distances based on the PL_K relation and on the W function. As a whole, we find that the weighted mean LMC and SMC distances based on theoretical relations are 18.51 ± 0.07 and 19.04 ± 0.09 respectively. On the other hand, the same distances based on empirical relations are: 18.40 ± 0.10 and 18.98 ± 0.10. This means that the two estimates agree at most within 5%. On the other hand, if we rely on the relations that are only marginally affected by reddening corrections, i.e. PL_K and Wesenheit function, we find that the mean distances are: 18.53 ± 0.08-19.04 ± 0.11 (theory) and 18.48 ± 0.13-19.01 ± 0.13 (empirical). Predicted and empirical distance do agree at the level of 2%.

Previous findings support the use of a noncanonical ML relation, and indeed theoretical predictions suggest that the zero-point of PL relations, in contrast with the slopes, do depend on the ML relation (Bono et al. 1999). It turns out that PL relations and W function based on pulsation models that assume a canonical ML relation (Marconi et al. 2002, in preparation) give distances ≈ 0.14 larger than previous ones. It is worth noting that predicted and empirical PL relations and Wesenheit function for FOs (Tables 2 and 3) do not depend, within the errors, on metal content.

3. Discussion and Conclusions

Figure 3 shows the comparison between theory and observations for LMC FO variables. Note that to investigate in detail the accuracy of current models we plotted the blue and the red edge of the instability region instead of mean relations. Data plotted in this figure do suggest that, within current uncertainties, predicted edges agree quite well with empirical data. The agreement found in the $M_I - \log P$ plane suggests that the discrepancy in the distance moduli based on the PL_I relation is due to a mild overestimate of individual reddening corrections. The dashed lines display the mean relations based on linear models provided by BA01. The comparison discloses that these relations marginally account for empirical data, since they are located close to blue edges of the instability strip. The difference between the two sets of pulsation models is mainly due to fact that BA01 adopted a canonical ML relation. Note that current FO edges do not show, in agreement with empirical data, a change in the slope when moving from 3.25 to 4 $M_\odot$ (Alibert et al. 1999).

The discrepancies we found are somehow at odds with the results obtained by BA01 concerning the difference between the slopes of their W functions and the slopes predicted by Caputo, Marconi, Musella (2000) for F pulsators. The main difference is that BA01 were forced to extrapolate the W function given by Caputo et al. (2000). In fact, these relations do rely on F models with periods ranging from $\log P = 0.50$ to $\log P = 1.86$ for Z=0.004 and from $\log P = 0.51$ to $\log P = 1.93$ for Z=0.008. However, both theoretical predictions (BCM01) and empirical data (Bauer et al. 1999; G00) support the evidence that the slope of the F edges changes when moving from higher to lower luminosities. This means that the extrapolation of PL and W relations toward shorter periods is risky, and therefore we did not perform any selection among unstable F and FO models.

The empirical lower limit in the period distribution of F and overtone Cepheids is a key observable to constrain the accuracy of pulsational and evolutionary models, since it supplies tight constraints on the minimum mass whose blue loop crosses the instability strip, as well as on the topology of the strip (Alcock et al. 1999; Bono et al. 2000; Beaulieu et al. 2001). On the other hand, the upper limit in the period distribution of FO Cepheids is a robust observable to constrain the plausibility of pulsation models (Bono et al. 1999; FBK00). FOs located close to the *intersection point*, i.e. the region of the instability strip where the blue and the red edge of FOs intersect, can be adopted to constrain the luminosity above which Cepheids only pulsate in the F mode. The longest predicted FO period is the aftermath of the topology of the instability strip, and in turn of the physical assumptions adopted to construct the pulsation models. According to empirical evidence based on the OGLE database the longest FO periods range from $\log P \approx 0.65$ to 0.77 for the SMC and from $\log P \approx 0.75$ to 0.80 for the LMC. Empirical estimates based on different Cepheid samples give quite similar upper limits for FOs (Beaulieu & Marquette 2000). Note that previous upper limits only bracket 4 (SMC) and 6 (LMC) FO Cepheids respectively. Moreover, the 4 SMC FOs with longer periods present a peculiar position in the Wesenheit plane, and therefore they could have been misclassified. These objects deserve a more detailed analysis to assess whether they are genuine FO pulsators. To avoid deceptive errors in the comparison between theory and observations we decided to adopt 0.65 and 0.75 as upper limits in the period cut-off.

Current predictions suggest that the cut-off periods for FO Cepheids in MCs are located at $\log P \approx 0.77$ (SMC) and $\log P \approx 0.73$ (LMC). The predicted SMC upper limit is ≈ 0.1 dex longer than observed, whereas the predicted LMC upper limit agrees quite well with the empirical one. Therefore it is not clear whether this discrepancy is due to a limit of theoretical models. It is noteworthy that pulsation models provided by BA01 predict stable FOs at periods longer than $\log P \approx 1.3$ (SMC) and $\log P \approx 1.4$ (LMC). These upper limits are substantially longer than observed ones. The reason for this difference is not clear. However, pulsation models based on a similar theoretical framework (linear, nonadiabatic), and treatment of convective transport, predict similar cut-off periods (Chiosi et al. 1993). To investigate the reasons of the mismatch between predicted and observed SMC FO cut-off periods we compared our predictions with empirical data for Galactic FO Cepheids. Empirical estimates suggest that the longest Galactic FO periods range from $\log P \approx 0.7$ to 0.88, but only 3 objects are included in this period range (Kienzle et al. 1999). Current models at solar chemical composition predict a cut-off period, $\log P \approx 0.6$, that is ≈ 0.1 dex shorter than the observed one. Note that nonlinear, convective models constructed by FBK00 predict an upper limit that is 0.25 dex longer ($\log P \approx 0.95$) than observed. Since these models rely on a similar theoretical framework the difference could be due to the different treatment in the turbulent convection model (see §4.1 in FBK00) and/or to the adopted ML relation. Synthetic Color-Magnitude diagrams that account for evolutionary and pulsation properties are mandatory to constrain star formation rates and/or theoretical predictions (Alcock et al. 1999).

The main conclusion we can draw is that current predictions concerning FO periods at the *intersection point* among Galactic and MC Cepheids agree within 0.1 dex with empirical values. A large sample of FO Cepheids in a different stellar system could be crucial to improve the accuracy of the empirical scenario. Accurate radius estimates of Galactic long-period FOs are strongly required to identify their pulsation mode. Different theoretical frameworks predict longer cut-off periods. This means that this observable can be adopted to validate the plausibility of pulsation models.

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REFERENCES

Alcock, C., et al. 1999, AJ, 117, 920

Alibert, Y. Baraffe, I. Hauschildt, P. & Allard, F. 1999, A&A, 344, 551

Baraffe, I., & Alibert, Y. 2001, A&A, 371, 592 (BA01)

Bauer, F. et al. 1999, A&A 348, 175

Beaulieu, J. P., Buchler, J. R., & Kolláth, Z. 2001, A&A, 373, 164

Beaulieu, J.-P., & Marquette, J.-B. 2000, in IAU Colloq. 176, The Impact of Large-Scale Surveys on Pulsating Star Research, ed. L. Szabados & D. Kurtz (San Francisco: ASP), 139

Bersier, D. 2000, ApJ, 543, L23

Bono, G., Caputo, F., Cassisi, S., Marconi, M., Piersanti, L., & Tornambè, A. 2000, ApJ, 543, 955

Bono, G. Caputo, F. Castellani, V., & Marconi, M. 1999, ApJ, 512, 711

Bono, G., Caputo, F., Marconi, M. 2001, MNRAS, 325, 1353 (BCM01)

Bono, G., Castellani, V., Marconi, M. 2002, ApJ, 565, L83

Bono, G., Gieren, W. P., Marconi, M., Fouqué, P., & Caputo, F. 2001, ApJ, 563, 319

Bono, G., Marconi, M., Stellingwerf, R. F. 1999, ApJS, 122, 167 (BMS99)

Caputo, F., Marconi, M., Musella, I. 2000, A&A 354, 610

Castelli, F., Gratton, R. G., & Kurucz, R. L. 1997, A&A, 324, 432

Chiosi, C., Wood, P. R., & Capitanio, N. 1993, ApJS, 86, 541

Feast, M. W., & Catchpole, R. M. 1997, MNRAS 286, L1

Feuchtinger, M., Buchler, J. R., & Kolláth, Z. 2000, ApJ, 544, 1056 (FBK00)

Girardi, L., Bressan, A., Bertelli, G., & Chiosi, C. 2000, A&AS, 141, 371

Groenewegen, M. A. T. 2000, A&A, 363, 901 (G00)

Groenewegen, M. A. T., Oudmaijer, R. D., 2000, A&A, 356, 849 (GO00)

Kienzle, F., Moskalik, P., Bersier, D., & Pont, F. 1999, A&A, 341, 818

Udalski, A. 1998, AcA, 48, 113

Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., Soszyński, I., Woźniak, P., & Żebruń, K. 1999, AcA, 49, 201

Udalski, A. Szymanski, M., Kubiak, M., Pietrzynski, G., Soszynski, I., Wozniak, P., & Zebrun, K. 1999, AcA, 49, 201

Table 1. Galactic relations of the form $M = a \times \log P + b$ using fixed theoretical slopes

S-M[a]	$a_{\rm theory}$	$b_{\rm theory}$	$b_{\rm Hipparcos}$	N[b]
		noncanonical models		
FO-W	-3.60 ± 0.04	-3.12 ± 0.03	-3.00 ± 0.14	27
FO-V	-3.01 ± 0.13	-1.66 ± 0.09	-1.65 ± 0.14	31
FO-I	-3.24 ± 0.09	-2.23 ± 0.06	-2.19 ± 0.16	27
FO-K	-3.52 ± 0.05	-2.87 ± 0.03	-2.86 ± 0.35	7
F-W	-3.12 ± 0.04	-2.79 ± 0.11	-2.87 ± 0.16	163
F-V	-2.27 ± 0.12	-1.37 ± 0.35	-1.97 ± 0.15	204
F-I	-2.60 ± 0.09	-1.93 ± 0.25	-2.34 ± 0.16	163
F-K	-3.09 ± 0.04	-2.52 ± 0.11	-2.72 ± 0.19	56
		canonical models		
FO-W	-3.75 ± 0.04	-3.26 ± 0.03	-2.92 ± 0.14	27
FO-V	-3.25 ± 0.19	-1.84 ± 0.15	-1.52 ± 0.14	31
FO-I	-3.44 ± 0.13	-2.40 ± 0.10	-2.08 ± 0.16	27
FO-K	-3.65 ± 0.07	-3.03 ± 0.05	-2.80 ± 0.35	7
F-W	-3.03 ± 0.03	-3.05 ± 0.10	-2.95 ± 0.16	163
F-V	-1.98 ± 0.08	-1.85 ± 0.25	-2.25 ± 0.15	204
F-I	-2.39 ± 0.06	-2.32 ± 0.19	-2.53 ± 0.16	163
F-K	-3.01 ± 0.03	-2.77 ± 0.10	-2.78 ± 0.18	56

[a]Predicted Wesenheit (W) and PL relations for First Overtone (FO) and Fundamental (F). [b] Number of objects used to derive the HIPPARCOS solution.

Table 2. MC distance moduli obtained using predicted FO PL-relations.

S-M[a]	Slope$_{\rm T}$[b]	ZP$_{\rm T}$[b]	ZP$_{\rm E}$[c]	DM[d]
		LMC		
1-W	-3.64 ± 0.03	-3.09 ± 0.04	15.45 ± 0.08	18.54 ± 0.09
2-I_0	-3.31 ± 0.07	-2.29 ± 0.09	16.13 ± 0.14	18.42 ± 0.17
3-K_0	-3.57 ± 0.03	-2.89 ± 0.05	15.62 ± 0.13	18.51 ± 0.14
		SMC		
4-W	-3.62 ± 0.03	-3.07 ± 0.04	15.99 ± 0.12	19.06 ± 0.13
5-I_0	-3.27 ± 0.07	-2.31 ± 0.09	16.67 ± 0.19	18.98 ± 0.21
6-K_0	-3.55 ± 0.03	-2.89 ± 0.05	16.13 ± 0.18	19.02 ± 0.19

[a]Solution. [b]Predicted slope and Zero-Point (ZP). [c]Empirical ZP, by fixing the slope to the theoretical one. Cuts are applied in period as follows: solutions 1 and 2 – none, solutions 3, 4 and 5 – $\log P > 0.25$, solution 6 – $\log P > 0.30$. [d]Distance modulus.

Table 3. MC distance moduli obtained using empirical FO PL-relations.

S-M[a]	$a_{\rm obs.}$[b]	$b_{\rm obs.}$[b]	$b_{\rm Hipparcos}$[c]	DM[d]
		LMC		
1-W	-3.40	15.373 ± 0.007	-3.11 ± 0.14	18.48 ± 0.14
2-I_0	-3.31	16.134 ± 0.001	-2.15 ± 0.16	18.28 ± 0.16
3-K_0	-3.39	15.540 ± 0.031	-2.91 ± 0.35	18.45 ± 0.35
		SMC		
4-W	-3.36	15.881 ± 0.033	-3.14 ± 0.14	19.02 ± 0.14
5-I_0	-2.87	16.520 ± 0.050	-2.39 ± 0.16	18.91 ± 0.17
6-K_0	-3.10	15.936 ± 0.068	-3.03 ± 0.35	18.97 ± 0.35

[a]Solution. [b] Observed FO PL-relation; W, K from G00, I-band derived here. Cuts are applied in period as follows: solutions 1 and 2 – none, solutions 3, 4 and 5 – $\log P > 0.25$, solution 6 – $\log P > 0.30$. [c] Galactic zero-point based on HIPPARCOS data for the observed slope. [d] Distance modulus.

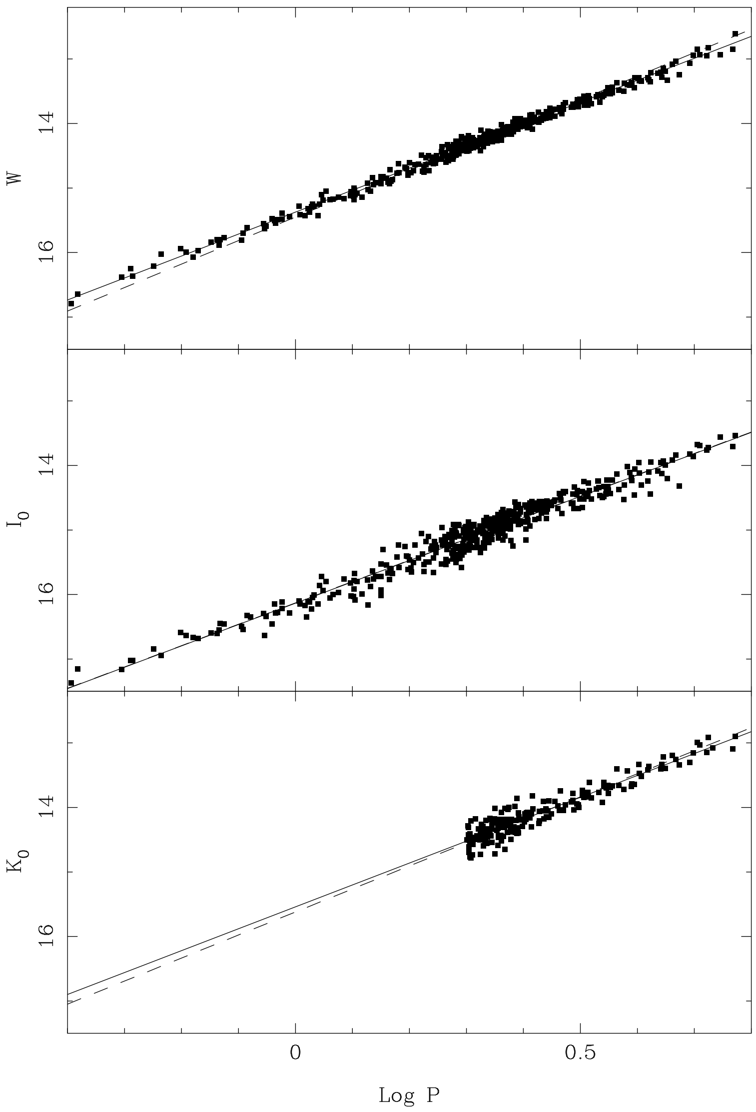

Fig. 1.— LMC, PL-relation in Wesenheit function, I_0 and K_0 (from top to bottom). Shown are the data (cut at $\log P = 0.3$ in K), the best fitting linear relation (solid line), and the relation determined by fixing the slope to its theoretical value.

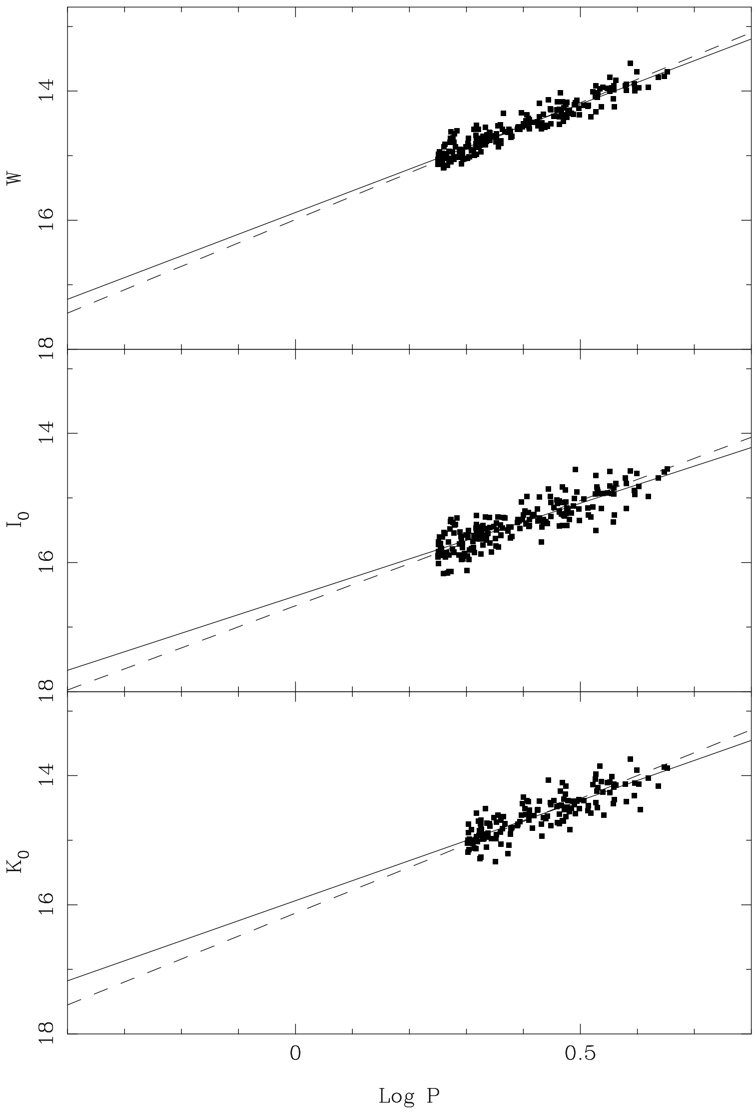

Fig. 2.— Same as Fig. 1, but for SMC Cepheids. The cut is at $\log P = 0.3$ (K), and 0.25 (W, I).

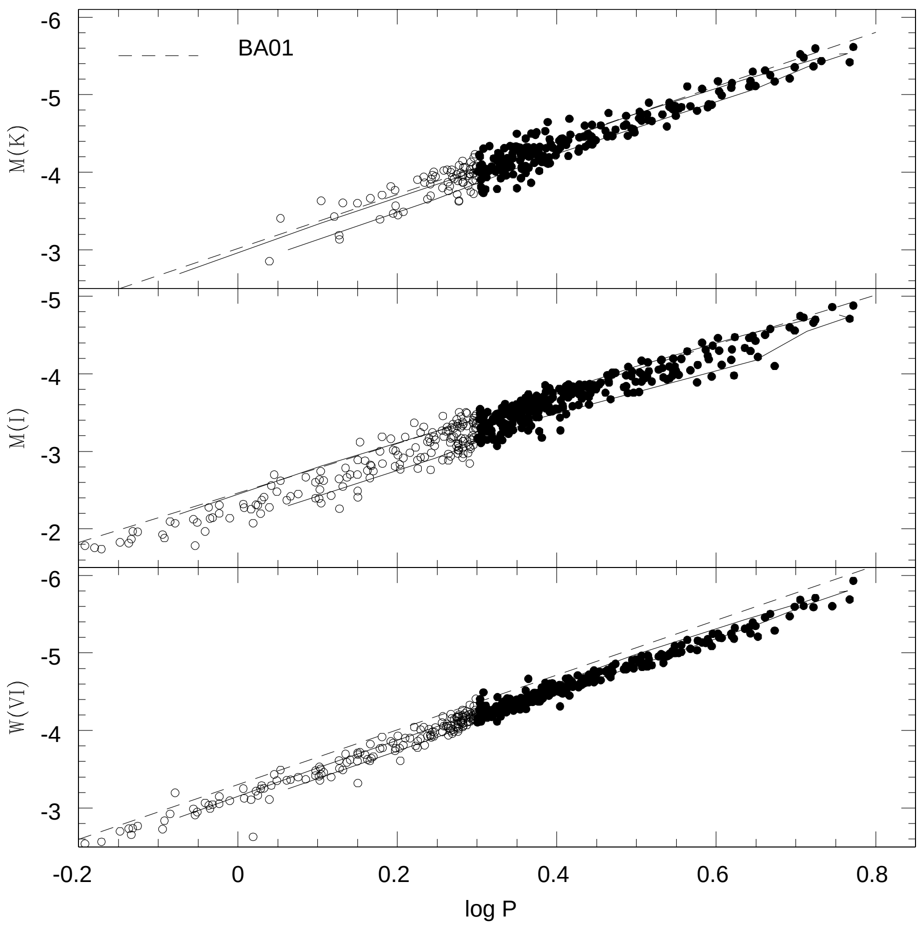

Fig. 3.— Comparison between predicted blue and red edges with LMC empirical data. Filled symbols show Cepheids adopted to estimate the distance. Dashed lines display mean PL relations provided by BA01. Predicted edges were plotted using the distances listed in Table 2.