# Synthetic AGB evolution

# IV. Long-period variables in the LMC

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**Abstract.** From observations of LPVs (long period variables) in the LMC we derive that the ratio of the number of C-rich LPVs to the total number of carbon stars is  $\sim$ 0.05 and that the ratio of the number of oxygen-rich LPVs to the total number of oxygen-rich AGB stars is between 0.05 and 0.10. The lifetime of the LPV phase in the LMC is only a few  $10^4$  yr, considerable below estimates for the Galaxy, where the duration of the LPV phase (about 2  $10^5$  yr) is similar to the total AGB lifetime. If the possible incompleteness of the surveys for LPVs is invoked to explain this discrepancy in the lifetimes then the ratio of small amplitude variables to large amplitude variables must be about 30, considerably larger than the ratio of Semi Regular to Mira variables in the Galaxy (1–3).

We present a simple model to explain the observed properties of LPVs in the LMC. It is assumed that pulsation only occurs in an instability strip in the HR diagram. The instability strip is characterised by three parameters: the temperature at some reference luminosity, the width of the instability strip and its slope  ${\rm d}T_{\rm eff}/{\rm d}M_{\rm bol}$ . The first two are free parameters in the model. Based on observations we use  ${\rm d}T_{\rm eff}/{\rm d}M_{\rm bol}=275~{\rm K~mag^{-1}}$  for  $M_{\rm bol}>-5$  and  $100~{\rm K~mag^{-1}}$  for  $M_{\rm bol}<-5$ . An additional complication is that the pulsation period depends rather sensitively on the effective temperature scale. The location of the AGB tracks in the HR diagram (the zero point of the effective temperature scale) is the third free parameter.

Both a model with a Reimers mass loss law inside and outside the instability strip, and a model with the mass loss in the instability strip given by a scaled version of the Blöcker & Schönberner (1993) mass loss law, fit the observational constraints equally well.

We conclude that first harmonic pulsation can be excluded unless the canonical relation between (J-K) color and effective temperature (based on lunar occultation observations) gives temperatures which are too high by  $\sim$ 20%, much larger than the estimated uncertainty of  $\sim$ 8% or possible systematic

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effects ( $\lesssim 10\%$ ). Fundamental mode pulsation is therefore probably the dominant pulsation mode among LPVs in the LMC.

A second conclusion is that for most stars the instability strip is not the final phase of AGB evolution. Based on our calculations for individual stars we find that AGB stars more massive than about  $1.2\,M_\odot$  spend a considerable amount of time in the phase between the end of pulsation and the end of the AGB.

We propose an alternative explanation for (some of) the non-variable OH/IR stars in the Galaxy.

**Key words:** stars: evolution of – stars: AGB – stars: variables: others – Magellanic Clouds

# 1. Introduction

Many AGB stars are observed to pulsate. However, the relation between the evolution of AGB stars of different masses and the different classes of variable stars [Miras, Semi-regulars (SRs) and Irregulars] and the evolution of the pulsation period remain uncertain. Recent studies (Jura & Kleinmann 1992a, b; Kerschbaum & Hron 1992) indicate that in the solar neighbourhood Miras and SRs with periods between 300 and 400 days have a scale height of about 250 pc, while Miras with periods between 100 and 300 days and SRs with periods between 200 and 300 days have a scale height of about 500 pc and thus have evolved from less massive progenitors. The situation for SRs with periods less than 200 days and for the Irregulars is less clear.

Hughes (1989) and Hughes & Wood (1990) have performed a deep and extensive search for long-period variables (LPVs) in the LMC. They found close to 1100 LPVs, about 470 showing large amplitude variations ( $\Delta I \geq 0.9$ , called Miras by them) and about 570 having smaller amplitudes (called SRs by them). Follow-on spectroscopy and near-infrared photometry has provided an indication of the C to M star ratio among the LPVs. The

survey was complete down to  $I \approx 18$ , equal to the completeness limit reached in optical surveys for AGB stars. For obvious reasons their survey is most sensitive to large amplitude variables. From their figures we deduce that the detection probability for a variable with an amplitude  $\Delta I = 1.2$  was close to 100%, but for a star with an amplitude of  $\Delta I = 0.6$  only  $\sim 50\%$ . Thus the presently known LPVs in the LMC contain essentially all Miras and SRs with large amplitudes. By using this well defined population of LPVs, for which luminosity, period and chemical type are relatively well known, we will in this paper attempt to place the LPVs in the general context of AGB evolution.

After a brief summary of our synthetic AGB evolution model (Sect. 2), the average duration of the LPV phase is derived from observations in Sect. 3. In Sect. 4 the observed period distribution of oxygen-rich and carbon-rich LPVs is fitted. We conclude in Sect. 5.

# 2. Synthetic AGB evolution

We have developed a model to calculate the evolution of AGB stars in a synthetic way (Groenewegen & de Jong 1993, Paper I). This model is more realistic than previous synthetic evolution models in that more details on the evolution both prior to, and on the AGB have been included. The variation of luminosity during the interpulse period was taken into account as well as the fact that, initially, the first few pulses are not yet at full amplitude and that the luminosity is lower than given by the standard core mass-luminosity relation. Most of the relations used are metallicity dependent. The model uses algorithms derived from recent evolutionary calculations for low- and intermediate-mass stars. The model is described in full detail in Paper I. Some essential aspects, relevant to this paper, are briefly introduced here.

In the model stars are selected according to their probability to be on the AGB, which depends on the star formation rate, the initial mass function and the lifetime on the AGB (see Paper I). We then calculate distributions of relevant quantities like the luminosity function, or, in this paper, the period distribution, for a population of stars.

The main free parameters of the model are the minimum core mass  $M_{\rm c}^{\rm min}$  for (third) dredge-up to occur, the dredge-up efficiency  $\lambda$  and the Reimers mass loss coefficient  $\eta_{\rm AGB}$ . In Paper I, mass loss on the AGB was described by a Reimers (1975) law:

$$\dot{M}_{\rm R} = \eta_{\rm AGB} \ 4.0 \ 10^{-13} \ \frac{L R}{M} \ {\rm M}_{\odot} {\rm yr}^{-1}$$
 (1)

where L,R and M have their usual meaning and are in solar units. In this paper we also consider the mass loss law derived by Blöcker & Schönberner (1993) based on the results of the dynamical modelling of LPVs by Bowen (1988):

$$\dot{M}_{\rm BS} = \eta_{\rm LPV} \ 4.8 \ 10^{-9} \ \frac{L^{2.7}}{M^{2.1}} \ \dot{M}_{\rm R} \ {\rm M}_{\odot} \ {\rm yr}^{-1}$$
 (2)

where  $\eta_{LPV}$  is a scaling factor, which is unity in Blöcker & Schönberner. The luminosity L is not the quiescent luminosity but includes the effect of the luminosity variation during the flashcycle, i.e. the mass loss rate just after a TP is higher than

during quiescent H-burning or in the luminosity dip. In Paper I we found that  $\eta_{AGB} \gtrsim 3$  is needed to fit the initial-final mass relation for the low mass stars and that  $\eta_{AGB} = 5$  provides the best fit to the high-luminosity tail of the carbon star luminosity function (LF). AGB evolution is terminated when the envelope mass is reduced to  $\sim 10^{-3} \, \mathrm{M}_{\odot}$ .

The third dredge-up process is incorporated as follows. Dredge-up operates only when the core mass is above a critical value  $M_{\rm c}^{\rm min}$ . In Paper I we found that  $M_{\rm c}^{\rm min}=0.58\,{\rm M}_{\odot}$  is needed to reproduce the low-luminosity tail of the carbon star LF. During dredge-up an amount of material

$$\Delta M_{\text{dredge}} = \lambda \, \Delta M_{\text{c}} \tag{3}$$

is added to the envelope, where  $\Delta M_c$  is the core mass growth during the preceding interpulse period. The composition of the dredged up material is assumed to be (Boothroyd & Sackmann 1988):  $X_{12}=0.22$  (carbon),  $X_{16}=0.02$  (oxygen) and  $X_4=0.76$  (helium). In Paper I we found that  $\lambda=0.75$  is needed to fit the peak of the carbon star LF. Hot bottom burning (HBB) has been included at the level of the Renzini & Voli (1981)  $\alpha=2$  case (see Appendix A of Paper I).

The effective temperature is calculated using the relations of Wood (1990) for AGB tracks in the HR diagram:

$$\log T_{\text{eff}} = (M_{\text{bol}} + 2.65 \log M)/15.7$$

$$-0.12 \log(Z/0.02) + \Delta + 3.764 \quad (M \le 1.5 \,\text{M}_{\odot})$$

$$= (M_{\text{bol}} + 4.00 \log M)/20.0$$

$$-0.10 \log(Z/0.02) + 3.705 \quad (M \ge 2.5 \,\text{M}_{\odot})$$
(4)

where  $M_{\rm bol} = -2.5 \log L + 4.72$  and  $\Delta$  is a correction term which accounts for the fact that the effective temperature increases at the end of the AGB phase when the envelope mass becomes small. The  $\Delta$ -term is calculated from Wood (1990):

$$\Delta = 0 \qquad x \ge 0.8$$
  
= 0.07 (0.8 - x)<sup>2.54</sup> x < 0.8  
$$x = M_{bol} + 7.0 - 1.2/M^{1.7}$$
 (5)

For stars with masses between 1.5 and 2.5  $\rm M_{\odot}$  we interpolate in  $\log T_{\rm eff}$  using the mass M as variable. The zero point of these relations was determined by Wood from the assumption that the star o Ceti (Mira) with a period of 330 days,  $\rm Z=Z_{\odot}$  and  $\rm M_{bol}=-4.32$ , has a mass of 1  $\rm M_{\odot}$  and is pulsating in the fundamental mode.

Fundamental mode and first harmonic pulsation periods are calculated as follows. The fundamental period (in days) is calculated following Wood (1990):

$$P_0 = 0.00851 R^{1.94} M^{-0.90} M \le 1.5 M_{\odot}$$
  
= 0.00363 R<sup>2.09</sup> M<sup>-0.77</sup> M \ge 2.5 M\_{\omega} (6)

Wood found this relation to be reasonably independent of metallicity. For stars with masses between 1.5 and  $2.5 \, \mathrm{M}_{\odot}$  we interpolate linearly in  $P_0$  using M as variable. The formalism to

calculate the first overtone period is adopted from Wood et al. (1983):

$$P_1 = Q R^{1.5} M^{-0.5} (7)$$

with

$$Q = 0.038 + 5.5 \cdot 10^{-5} (P_1 - 100) \quad M \le 0.85$$
and  $P_1 \ge 100$ 

$$= 0.038 + 4.5 \cdot 10^{-5} (P_1 - 150) \quad 0.85 < M \le 1.5$$
and  $P_1 \ge 150$ 

$$= 0.038 + 2.5 \cdot 10^{-5} (P_1 - 300) \quad 1.5 < M \le 2.5$$
and  $P_1 \ge 300$ 

$$= 0.038 \quad \text{all other cases}$$

Equations (4–8) have been derived for oxygen-rich stars. Lacking any better estimate we will also use them for carbon stars. This assumption is discussed in Sect. 5.

In Sect. 4 we derive effective temperatures from observations for both oxygen-rich and carbon-rich stars from (Bessell et al. 1983):

$$T_{\rm eff} = \frac{7070}{(J - K) + 0.88} \tag{9}$$

where the (J-K) color is in the Johnson system. This relation has been calibrated using effective temperature determinations from the lunar occultation observations of Ridgway et al. (1980a, 1980b). The accuracy of Eq. (9) is about 250 K. It is possible however that Eq. (9) gives too low effective temperatures for the carbon stars or that there is a systematic effect in applying this empirical equation, derived from stars in the solar neighbourhood, to LMC stars.

#### 3. The duration of the LPV phase

In their study of LPVs in the LMC, Hughes (1989) and Hughes & Wood (1990) identified 594 definite and 449 probable LPVs in an 53 deg<sup>2</sup> area. Of the definite LPVs, 247 showed large amplitude variations in their lightcurves ( $\Delta I \geq 0.9$ , called Miras) and 347 showed smaller variations ( $\Delta I < 0.9$ , called Semi-Regulars). Of the 449 probable LPVs, 224 showed Mira-like behaviour and 225 SR-like behaviour.

About 500 stars were classified as carbon- or oxygen-rich based on low resolution spectra or (J-K) color. Of 307 Miras, 119 were classified as carbon stars (38.8%), of 181 SRs investigated, 69 were classified as carbon stars (38.1%). Extrapolating to the total number of 1043 LPVs we derive an estimated number of about 400 carbon-rich and about 640 oxygen-rich LPVs. The same area in the LMC contains about 7500 carbon stars and between 6700 and 12000 oxygen-rich AGB stars (Paper I).

The fraction of LPVs among AGB stars is 0.053 for the carbon-rich and between 0.054–0.096 for the oxygen-rich AGB stars. Using the average lifetimes of the AGB phase for O-rich and C-rich species (from Paper I), this corresponds to a mean lifetime of the carbon-LPV and oxygen-LPV phase of  $\sim 1.1 \, 10^4$  and 0.7–1.8  $10^4$  yrs respectively. Based on the observed C/M

ratio of 0.63 in the LPV phase an independent estimated lifetime for the oxygen-rich LPV phase of 1.8 10<sup>4</sup> yrs is derived.

An independent estimate of the lifetime of the LPV phase can be made from Renzini & Buzzoni's (1986) fuel consumption theorem,  $N_i = B(t) L_T t_i$ . This relates the total number of objects  $N_i$  in a specific evolutionary phase to the bolometric luminosity  $L_{\rm T}$  and the stellar death rate per unit luminosity B(t). The apparent blue magnitude of the LMC is 0.1 (Biermann 1982). With a distance modulus of 18.5 (Panagia 1991) and a absolute blue magnitude of the Sun of 5.41 we derive a blue luminosity of the LMC of about  $3.3 \times 10^9 \, L_{\odot}$ . This corresponds to a bolometric luminosity of about ~ 3 larger (cf. Renzini 1993) or  $1.0 \, 10^{10} \, \text{L}_{\odot}$ . With  $B(t) = 2 \, 10^{-11} \, \text{stars yr}^{-1} \, \text{L}_{\odot}^{-1}$  (Renzini & Buzzoni 1986) and  $N_i = 1000$  this results in a lifetime of the LPV phase of 5  $10^3$  yr, if the whole population evolves through the LPV phase. Since this estimate is lower than the previous estimate of a few 10<sup>4</sup> yr, this last condition is probably not fulfilled.

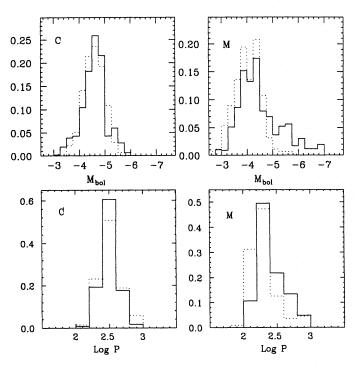
The duration of the LPV phase of a few  $10^4$  yr is well below the estimates for galactic LPV populations. Recently Jura & Kleinmann (1992) derived a lifetime of  $2\,10^5$  yr for Miras in the solar neighbourhood with 300 < P < 400 days based on the death rate of their main-sequence progenitors. Estimates using the fuel consumption theorem give lifetimes of at least  $10^5$  yr for Miras in the Bulge (Whitelock 1993),  $2.5\,10^5$  yr for globular cluster Miras (Renzini & Greggio 1990) and  $2.5\,10^5$  yr for LPVs in 47 Tuc (Renzini 1993). This can be compared to the average AGB lifetime of about  $3\,10^5$  yr from recent synthetic AGB calculations (Groenewegen et al. 1994). For the Galaxy, stars seem to be LPVs during most of their AGB phase, while for the LMC this seems not to be the case.

The reason why LPVs in the Galaxy and in the LMC appear to have different lifetimes is unknown at present. Alternatively, one could invoke the possibility that the surveys for LPVs have been incomplete by a factor of about 15 to reconcile the differences in the lifetimes. Since one can assume the surveys to be complete for large amplitude variables this implies that the ratio of the small amplitude variables (the about 15000 LPVs yet undiscovered + 570 known SRs) to that of the large amplitude variables (the 470 Miras known) would be about 30. For the Galaxy the ratio of SRs to Miras is 1–3 (Jura & Kleinmann 1992; Kerschbaum & Hron 1992). In the presently known population of LPVs in the LMC the ratio of small to large amplitude variables is about 1.2, close to the value in the Galaxy.

In short, LPVs in the LMC seem to behave quite differently from those in the Galaxy. Either their lifetime is much shorter (if the present surveys for LPVs in the LMC are reasonably complete) or the ratio of small to large amplitude variables is much larger than in the Galaxy (if the present surveys are incomplete by a factor of 15).

#### 4. Fitting the period distribution of LPVs

In Fig. 1 the observed LFs and period distributions of carbon and oxygen-rich Miras (the solid line) and SRs (the dotted line) are plotted. The oxygen-rich SRs are concentrated towards slightly



**Fig. 1.** The observed luminosity function and period distribution of carbon and oxygen-rich 'Miras' and 'Semi Regulars' (SRs) variables (as defined by Hughes 1989) in the LMC from the data of Hughes (1989) and 7 Hughes & Wood (1990). The Miras are represented by the solid lines, the SRs by the dotted line. There is a tendency (in particular for the oxygen-rich stars) for the SRs to have lower luminosities and lower periods than the Miras. All histograms are normalised to unity

lower luminosities and lower pulsation periods. For the carbonrich LPVs the difference between Miras and SRs is even smaller. In the remainder of this section we will not distinguish between Miras and SRs and we will add the observed period distribution (weighted by number) to obtain the observed period distribution of LPVs in the LMC.

We first calculated the fundamental and first harmonic pulsation period distributions for the standard model of Paper I according to Eqs. (6–8), under the assumption that stars pulsate everywhere on the AGB. The resulting period distributions are compared to the observed one in Fig. 2. There is strong disagreement. Both the fundamental and the first harmonic period distributions are too broad, i.e. the model predicts pulsation at both too low and too high periods. This is true for both oxygenrich and carbon stars.

In Fig. 3 we show the evolution of two stars of  $1.25\,M_\odot$  (O) and  $5\,M_\odot$  (X) in the period-luminosity (P-L) diagram. The variation of the luminosity during the flashcycle is represented by a block profile (Paper I). This is reflected in Fig. 3 where stars jump from one phase to another: the luminosity dip, quiescent H-burning and the shell flash. The assumption that pulsation occurs everywhere on the AGB results in periods which are both lower and higher than observed (taken from Hughes & Wood 1990; the strip bounded by the two full lines in Fig. 3). This is

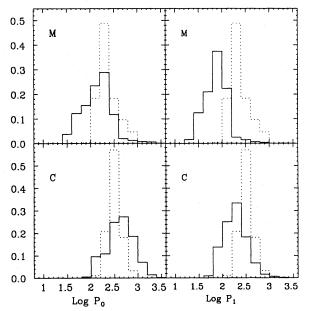


Fig. 2. The predicted (solid lines) fundamental mode  $(P_0)$  and first harmonic  $(P_1)$  pulsation period distribution for the standard model of Paper I calculated under the assumption that stars pulsate during their entire AGB life. The dotted line represents the observed period distribution of LPVs in the LMC. All histograms are normalised to unity

true for both low and high initial masses. A similar conclusion is derived by Vassiliadis & Wood (1993).

The naive assumption that AGB stars always pulsate is incorrect. The LPV phase is, on average, a brief one ( $\sim$ 6% of the total AGB phase). Thus, either all AGB stars go through a brief LPV phase, or, only a small fraction of AGB stars is LPV during their entire AGB life. There are several arguments to favor the first hypothesis, i.e. a majority of AGB stars going through a brief LPV phase. Firstly, as shown in Figs. 2 and 3, any prolonged LPV phase results in period distributions which are too broad. Secondly, LPVs are observed over a wide range of masses, from low mass stars in Galactic globular clusters (Menzies & Whitelock 1985) to the more massive OH/IR stars. If most intermediate mass stars can become a LPV then each star can only be a LPV for a short time interval.

We consider an instability strip of the form:

$$T_{l} = T_{l}^{0} + \frac{dT_{eff}}{dM_{bol}}(M_{bol} + 5)$$

$$T_{h} = T_{h}^{0} + \frac{dT_{eff}}{dT_{eff}}dM_{bol}(M_{bol} + 5)$$
(10)

where  $T_1$  and  $T_h$  are the low- and high-end effective temperatures of the instability strip. The width of the instability strip  $(T_h - T_1)$  determines the overall duration of the LPV phase, while the position of the instability strip in the HR diagram determines the C/M ratio in the instability strip. The free parameters are  $T_1^0$  and  $T_h^0$ . The third free parameter is the zeropoint of the effective temperature scale [cf. Eq. (4)] that fixes the location of the AGB tracks in the HR diagram. The pulsation periods are sensitive

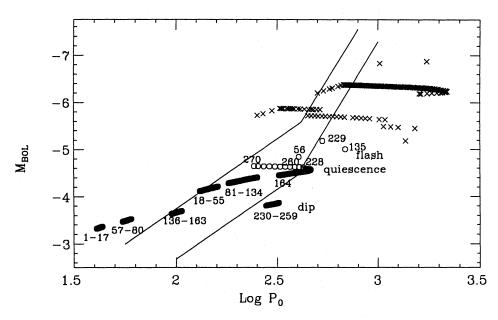


Fig. 3. The (fundamental mode) period-luminosity relation for stars of  $1.25 \, \mathrm{M}_\odot$  (O) and  $5 \, \mathrm{M}_\odot$  (X) for the standard model of Paper I calculated under the assumption that pulsations occurs during the entire AGB. The  $P_1 - L$  relation is qualitatively similar, but for a given  $M_{bol}$  the tracks are shifted by about 0.3-0.4 dex to lower periods. The observed period range for a given luminosity is given by the two full lines (from Hughes & Wood 1990). The time evolution of the  $1.25 \, \mathrm{M}_\odot$  model is indicated (lifetimes in  $10^3 \, \mathrm{yr}$ ). The interval between points plotted is 1000 years. The evolution of the  $5 \, \mathrm{M}_\odot$  model is similar. Because the luminosity variation during the interpulse period was assumed to be a block profile, stars jump from the luminosity dip to the quiescent H-burning phase to the thermal flash

to the stellar radius [Eqs. (6–8)] and hence the effective temperature. The value of  $\eta_{\rm AGB}$  has to be modified compared to the value in the standard model when the zero point of the effective temperature scale is changed, to give identical evolutionary behaviour on the AGB ( $\dot{M} \sim \eta_{\rm AGB} R \sim \eta_{\rm AGB} T_{\rm eff}^{-2}$ ).

The slope  $\mathrm{d}T_{\rm eff}/\mathrm{d}M_{\rm bol}$  can be determined from observations. Feast et al. (1989) have derived a relation between (J-K) color, averaged over the lightcurve, and  $\log P$  for oxygen-rich Miras in the LMC. Combining this data with Eq. (9) and with the mean P-L-relation of Hughes & Wood (1990) we derive  $\mathrm{d}T_{\rm eff}/\mathrm{d}M_{\rm bol} \approx 275~\mathrm{K~mag^{-1}}$  for  $M_{\rm bol} > -5$  and  $\approx 100~\mathrm{K~mag^{-1}}$  for  $M_{\rm bol} < -5$ . We assume this relation to hold also for SR variables. For carbon-rich LPVs we also assume this relation to hold because the scatter in the relation between (J-K) color and  $\log P$  for carbon stars is very large (Feast et al. 1989).

The constraints to the model are the duration of the LPV phase relative to the total AGB phase for both oxygen-rich and carbon-rich stars (see Sect. 3), the observed pulsation period distribution for both oxygen-rich and carbon-rich stars (the dotted histogram in Fig. 2), and the effective temperatures of Miras in the LMC derived from Eq. (9) and the (J-K) colors in Feast et al. (1989). Later on we will also discuss the ability of the models to reproduce the observed  $\dot{M}-P$  relation for galactic stars. Based on the results of Sect. 3, a duration of the C-star LPV phase relative to the carbon star AGB phase of 0.053 and a C/M ratio in the instability strip of 0.63 are used as constraints.

It should be emphasized that when we refer to the effective temperature of LPVs, we implicitly assume the effective temperature of a non-pulsating star. The pulsation will trigger

variations in the effective temperature resulting in real LPVs to have effective temperatures which may be outside the instability strip.

The fitting procedure is as follows. We consider four zero points of the effective temperature scale: the original zero point of Paper I (Eq. 4) and zero points lower by 0.02 dex, 0.05 dex and 0.10 dex. The value of  $\eta_{AGB}$  has to be modified when the zero point of the effective temperature scale is changed, as discussed before, in particular,  $\eta_{AGB}=5.0, 4.6 \ 4.0, 3.15$  are used, respectively. For the moment  $\eta_{AGB}$  is assumed to be equal inside and outside the instability strip. In the program, pulsation periods are calculated for stars in the instability strip, i.e. when  $T_1 \leq T_{\rm eff} \leq T_{\rm h}$ . The temperatures  $T_1^0$  and  $T_{\rm h}^0$  are chosen such that fit the assumed duration of the carbon star LPV phase and the C/M ratio in the instability strip. The predicted period distribution for both M- and C-stars is then compared to the observed period distribution.

The results of the calculations are shown in Fig. 4. For  $\Delta \log T_{\rm eff} = 0, -0.02, -0.05, -0.10$  relative to the zero point adopted in Paper I, we find  $T_{\rm l}^0 = 3330, 3180, 2970, 2640$  K and  $T_{\rm h}^0 = 3380, 3227, 3014, 2682$  K respectively. The  $\Delta \log T_{\rm eff} = -0.02$  model for fundamental mode pulsation provides the best overall fit. The predicted effective temperatures of LPVs at  $M_{\rm bol} = -5$  ( $T_{\rm eff} \approx 3200$  K) is in agreement with the observed value ( $T_{\rm eff} = 3180$  K) derived from the data in Feast et al. (1989). At other luminosities the agreement is equally good since the slope  ${\rm d}T_{\rm eff}/{\rm d}M_{\rm bol}$  is not a free parameter but determined by observations. When the period distribution for M- and C-stars are considered separately a  $\Delta \log T_{\rm eff} \approx -0.01$  model would best

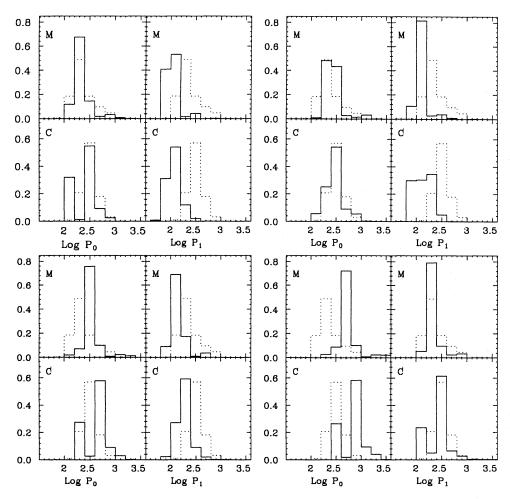


Fig. 4. The calculated fundamental mode  $(P_0)$  and first harmonic  $(P_1)$  pulsation periods (the solid lines), compared to the observed period distribution (the dotted lines). The periods are calculated under the assumption that pulsation only occurs in an instability strip in the HR diagram, bounded by the temperatures  $T_1$  and  $T_h$ , given by Eq. (10). These calculations are performed for the zero point of the effective temperature scale of Paper I (top left panel), a zero point lowered by 0.02 dex (top right), lowered by 0.05 dex (bottom left) and lowered by 0.1 dex (bottom right). The values of  $T_1^0$  and  $T_h^0$  are given in the text. The mass loss rate law is given by Eq. (1). All histograms are normalised to unity

fit the M-star period distribution while a  $\Delta \log T_{\rm eff} \approx -0.03$  model would best fit the carbon star period distribution. The small remaining discrepancy between the observed and the predicted period distributions may be due to uncertainties in the pulsation constants or a difference in pulsation constants between carbon- and oxygen-rich stars.

A reasonable fit to the observed period distributions could also be achieved for the first harmonic pulsation mode if  $\Delta \log T_{\rm eff} \approx -0.12$ . The predicted effective temperature at  $M_{\rm bol} = -5$  would be  $\sim\!2550\,\rm K$ . This would imply that Eq. (9) gives temperatures too high by  $\sim\!20\%$ , much larger than the quoted uncertainty which is  $\sim\!8\%$ . Based on these arguments we favor fundamental mode pulsation as the (dominant) mode of pulsation in LPVs in the LMC.

In our calculations we adopted a Reimers law in the instability strip. There is observational evidence that in LPVs pulsation and mass loss are related (De Giola-Eastwood et al. 1981; Schild 1989; Wood 1990; Whitelock 1990). We therefore also consider

a mass loss rate in the instability strip which is based on Blöcker & Schönberner's (1993) fit to the modelling of LPVs by Bowen (1988). Outside the instability strip we keep the Reimers mass loss rate Eq. (1). We performed some test calculations to determine the scaling factor  $\eta_{LPV}$  (cf. Eq. 2) since the absolute values of the mass loss rates derived by Bowen are uncertain due to uncertainties in his model. Furthermore, the mass loss rates and effective temperatures of LPVs in the LMC and in the Galaxy may be different. We proceeded as follows. A value for  $\eta_{LPV}$  was assumed. We first determined the value of the mass loss scaling parameter outside the instability strip ( $\eta_{AGB}$ ) by fitting the observed carbon star luminosity function and observed the C/M ratio of AGB stars (see Paper I). As before, we then optimised the fit to the period distribution by modifying the zero point of the effective temperature scale. We calculated the  $\dot{M}-P$  relation for some stars and guessed a new value for  $\eta_{\rm LPV}$ . The model which best fits the period distribution of the M-stars has the following parameters:  $\Delta \log T_{\rm eff} = 0.0$ ,  $\eta_{\rm LPV} = 0.055$ ,

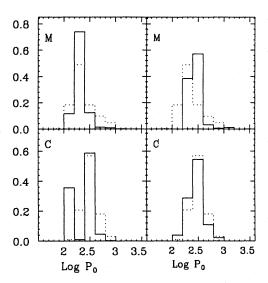


Fig. 5. The calculated fundamental mode  $(P_0)$  pulsation period distribution (the solid lines), compared to the observed period distribution (the dotted lines). The periods are calculated under the assumption that pulsation only occurs in an instability strip in the HR diagram, bounded by the temperatures  $T_1$  and  $T_h$ , given by Eq. (10). The zero point of the effective temperature scale is equal to (left part) and 0.02 dex lower than the scale used in Paper I (right part). The mass loss scaling parameter outside the instability strip,  $\eta_{AGB}$ , is 5.15 (left side) and 4.7 (right side). The mass loss scaling parameter inside the instability strip,  $\eta_{LPV}$ , is 0.055 (left side) and 0.05 (right side). All histograms are normalised to unity

 $\eta_{\rm AGB}=5.15,~T_1^0=3350~{\rm K},~T_{\rm h}^0=3390~{\rm K}.$  The model which best fits the period distribution of the C-stars has the following parameters:  $\Delta\log T_{\rm eff}=-0.02,~\eta_{\rm LPV}=0.05,~\eta_{\rm AGB}=4.7,~T_1^0=3175~{\rm K},~T_{\rm h}^0=3210~{\rm K}.$  The (fundamental mode) period distributions for both models are shown in Fig. 5. They fit the observed distribution equally well as the simpler model where the mass loss is equal in and outside the instability strip. The fact that  $\eta_{\rm LPV}\ll\eta_{\rm AGB}$  may give the false impression that mass loss in LPVs is insignificant. This is an artifact of the definitions of the mass loss rate formulas, Eqs. (1) and (2). Direct calculation, or Fig. 6 discussed below, illustrates the typical mass loss rates in the LPV phase.

In Fig. 6 the  $\dot{M}-P$  relation is shown for both the Reimers mass loss model and for the two-component mass loss model (both with  $\Delta \log T_{\rm eff} = -0.02$ ). For comparison we show the observed relations in the Galactic bulge (Whitelock 1990) and the solar neighbourhood (Schild 1989 and Wood 1990). The error in the observed relations is about 0.2-0.5 dex in  $\dot{M}$  for a given pulsation period. This uncertainty is larger than the possible systematic effect of comparing a model for the LMC with observations for the Galaxy. If  $\dot{M} \sim Z^{0.5}$ , as derived by Kudritzki et al. 1987 for massive stars, then there is a systematic shift of about 0.2 dex in  $\dot{M}$ . The relation of Wood is in disagreement with those of Schild and Whitelock, which suggests that the AGB lifetimes of the low mass stars derived by Vassiliadis & Wood (1993) have been overestimated. The slope in the  $\dot{M}-P$  relation is well fitted for the BS mass loss law in the instability

strip. This is due to the  $L^{3.7}$  dependence of the mass loss rate. With a Reimers law ( $\sim L$ ) the slope in the  $\dot{M}-P$  relation can not be reproduced as well.

#### 5. Discussion and conclusions

Our exploratory quantitative study into the pulsational properties of LPVs in the LMC leads to two conclusions: (1) fundamental mode pulsation is the (dominant) pulsation mode of LPVs in the LMC, and (2) for most AGB stars the instability strip where (large amplitude) pulsation occurs is not the final phase of AGB evolution.

The mode of pulsation of LPVs has long been a point of controversy. Recently, a consensus seems to have been reached in favor of fundamental mode pulsation (Hill & Willson 1979; Bowen 1988; Wood 1990), however see Tuchman (1991). Our results suggest fundamental mode pulsation as the dominant pulsation mode. Based on our model we exclude first harmonic pulsation unless Eq. (9) would overestimate the effective temperature by  $\sim$ 20%, which is much larger than the quoted uncertainty of  $\sim$ 8%. Equation (9) was derived for Galactic stars but has traditionally been used for the LMC as well. Based on Eq. (4) we estimate that for fixed mass and luminosity a star in the LMC has a 8–10% lower effective temperature than a corresponding star in the Galaxy.

We implicitly assumed that all LPVs found by Hughes are (thermal pulsing-) AGB stars and not, for example, early-AGB stars. For most of the carbon stars this assumption is likely to be valid. Only for the lowest luminosity bins there may be a contribution by non-TP AGB stars (see the discussion in Paper I). When the observed LF of oxygen-rich LPVs is compared to the (predicted) LF of oxygen-rich AGB stars (Fig. 7) one sees that the two LF almost overlap. If the LPV population would contain a significant number of low-luminosity early-AGB (E-AGB) stars one would expect that the LF of LPVs would be more concentrated towards low luminosities, especially since the E-AGB phase lasts much longer than the TP-AGB phase. We conclude that most LPVs found in the Hughes survey are indeed (TP-) AGB stars.

Independent of the exact assumptions on the shape of the instability strip and the mass loss rate in and outside the instability strip, AGB evolution does not end in the instability strip for most stars. This is emphasized in Table 1 where some characteristic lifetimes have been listed for individual stars for the models with  $\eta_{AGB}=4.6$  and  $\eta_{AGB}=4.7$ ,  $\eta_{LPV}=0.05$  (in both cases  $\Delta \log T_{eff}=-0.02$ ). Stars more massive than about  $1.2\,\mathrm{M}_\odot$  spend a considerable amount of time to the right (in the HR diagram) of the instability strip and stars below about  $1.15\,\mathrm{M}_\odot$  do not reach the instability strip. Although this is contrary to the widespread belief that AGB evolution ends when the star is an LPV, the observations of LPVs in the LMC point to a different conclusion.

Hughes et al. (1991) derived from a kinematical study that the oldest LPVs (those with periods between 100 and 225 days) seem to have ages of about 9.5 Gyr. Interpolating in the recent evolutionary tracks of Vassiliadis & Wood (1993) for Z = 0.008

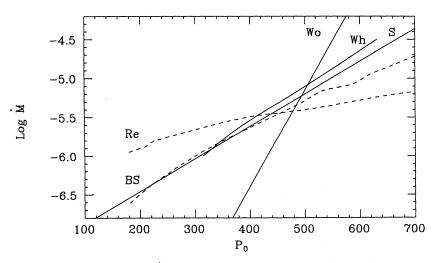


Fig. 6. The relation between  $\dot{M}$  and fundamental mode pulsation period  $P_0$  for the model with a Reimers law in and outside the instability strip ( $\Delta \log T_{\rm eff} = -0.02$  dex,  $\eta_{\rm AGB} = 4.6$ ) indicated by 'Re' and the model with the BS law in the instability strip ( $\Delta \log T_{\rm eff} = -0.02$  dex,  $\eta_{\rm AGB} = 4.7$ ,  $\eta_{\rm LPV} = 0.05$ ) indicated by 'BS'. Also shown are the observed relations in the Galactic bulge (Whitelock 1990, 'Wh') and the solar neighbourhood (Schild 1989, 'S', and Wood 1990, 'Wo')

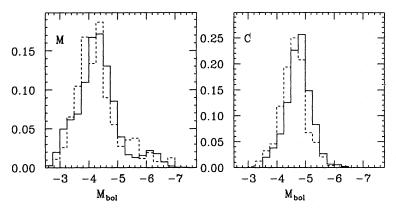


Fig. 7. In the left panel the theoretically predicted luminosity function (LF) of oxygen-rich AGB stars (solid line) is compared to the observed LF of oxygen-rich LPVs (dotted line). In the right-hand panel the observed LF of carbon stars on the AGB (solid line) is compared to the observed LF of carbon-rich LPVs (dotted line). The histograms are normalised to unity

this corresponds to stars with initial masses of about  $1.05\,M_\odot$ , close to our derived lower limit of  $1.15\,M_\odot$ . It would be important to determine this lower mass limit more precisely as it would provide an additional constraint to the model and could fix the location of the instability strip in the HR diagram more precisely.

In the Galaxy there is the class of the non-variable OH/IR stars (Habing et al. 1987). If a similar scenario holds for the Galaxy as we derive for the LMC, the non-variable OH/IR stars can be interpreted as massive stars ( $M_{\rm initial} \gtrsim 3.5 \, {\rm M}_{\odot}$ ) which are now in the phase between the end of the instability strip and the end of the AGB. This is an alternative explanation to the one proposed by Habing et al. (1987). Based on the fact that the spectrum of (some) non-variable OH/IR stars is redder than normal OH/IR stars, suggesting that the inner radius of the dust shell has moved away from the star (due to a lower present-day mass loss) Habing et al. suggested that the non-variable OH/IR stars are in the process of moving from the AGB to the post-

AGB phase. In our scenario the drop in the mass loss rate is due to the transition from the high-mass-loss instability strip to the lower-mass-loss final AGB phase.

Van der Veen (1989) showed that stars in region IV and V of the IRAS color-color diagram with energy distributions similar to the non-variable OH/IR stars originate from  ${\sim}4\,M_{\odot}$  stars and have present-day mass loss rates of  $10^{-6}\text{--}10^{-5}\,M_{\odot}\,\text{yr}^{-1},$  surprisingly high for post-AGB mass loss which is typically only  $10^{-8}\text{--}10^{-6}\,M_{\odot}\,\text{yr}^{-1}.$ 

The transition from the AGB to the post-AGB phase is a brief one ( $\sim 10^3$  years, see e.g. Slijkhuis 1992). The Habing et al. scenario cannot explain why so many of the OH/IR stars are non-variable (van Langevelde 1992 finds that  $\sim 20\%$  of OH/IR stars in the Galactic center are non-variable). In our scenario the non-variable OH/IR phase lasts about  $10^4-10^5$  years relative to a total AGB phase of  $1-5\ 10^5$  years (see Table 1). If these lifetimes also apply to the Galaxy we predict about 10-20% non-variable OH/IR stars, in reasonable agreement with observations.

Table 1. Results for some masses

$\overline{M}$	$Z^{-}$	$T_{M}$	$T_{S}$	$T_{\rm C}$	$T_{AGB}$	$T_{ m before}$	$T_{in}$	$T_{ m after}$
0.93	0.0020	230	-	-	230	230	-	
		227		-	227	227	-	
0.96	0.0028	214	-	-	214	214	-	- j <del>-</del>
		211	-	-	211	211	-	-
1.00	0.0037	159	-	-	159	159	-	-
		157	-	-	157	157	. <b>-</b>	-
1.18	0.0061	140	-	91	231	64	36	131
		139	-	96	235	77	23	136
1.30	0.0068	131	92	62	285	75	32	179
		131	92	72	295	89	51	156
1.50	0.0076	124	88	156	368	111	42	215
		124	88	161	373	123	36	214
2.00	0.0082	272	-	338	610	255	44	310
		272	_	347	619	269	35	315
2.50	0.0084	329	84	469	882	465	56	362
		329	84	487	900	477	64	360
3.00	0.0086	380	82	618	1080	658	37	385
		302	160	626	1088	662	55	370
3.50	0.0086	142	55	226	423	212	19	192
		142	37	230	409	213	9	187
4.00	0.0087	219	48		267	108	21	138
		208	38	-	246	108	8	130
5.00	0.0087	189	-	-	189	60	16	113
		167	-	-	167	59	5	104

*Notes.* Listed are the initial mass (in  $M_{\odot}$ ), the metallicity Z, the lifetimes of the M, S, C and the total AGB phase, the lifetimes before, in and after the instability strip (in  $10^3$  years). The first line is for the model with  $\eta_{AGB} = 4.6$ , the second line for  $\eta_{AGB} = 4.7$ ,  $\eta_{LPV} = 0.05$ . In both cases is  $\Delta \log T_{\rm eff} = -0.02$ .

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