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A revised period–luminosity relation for carbon Miras

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ABSTRACT

The period–luminosity and period–*K* magnitude relations of carbon Miras in the Galaxy are derived. The procedure is a two-step process. First, the *P–L* and *P–K* relations of carbon Miras in the Large Magellanic Cloud (LMC) are derived from 54 spectroscopically confirmed carbon Miras with periods between 150 and 520 d. The slopes of these relations are assumed to hold for Miras in the Galaxy, an assumption which has proved valid for oxygen-rich Miras. Secondly, the zero-points of the Galactic *P–L* and *P–K* relations are determined by shifting the relations for the LMC Miras, assuming a distance modulus of 18.5 mag, and no correction for metallicity differences.

An (uncertain) estimate for the zero-point of the *P–L* relation is derived by combining the observed peak of the period distribution of Galactic carbon Miras with the theoretically predicted peak of the carbon star luminosity function. A direct estimate is also made of the luminosity of the only carbon Mira known to have a normal binary companion. The zero-points derived by these two methods are consistent with that of shifting the LMC relation. The finally adopted *P–L* and *P–K* relations of Galactic carbon Miras are $M_{\rm bol} = -2.59 \log P + 2.02$ and $M_K = -3.56 \log P + 1.14$.

Key words: stars: AGB and post-AGB – stars: carbon – circumstellar matter – stars: mass-loss – stars: variables: other – infrared: stars.

1 INTRODUCTION

The period–luminosity (*P–L*) and period–*K* magnitude (*P–K*) relations are useful empirical tools in deriving distances to Mira variables. Most of the work until now has concentrated on oxygen-rich Mira variables.

Feast et al. (1989) derived *P–L* and *P–K* relations for carbon Miras in the LMC, valid for stars with periods *P* less than 420 d. The relation they derived was based on multiple observations of 20 stars, of which 10 were spectroscopically confirmed carbon stars and 10 were assumed to be carbon stars because of their location in a ($J-H$, $H-K$) colour–colour diagram. Since then, Hughes (1989), Hughes & Wood (1990) and Reid, Hughes & Glass (1995) have identified large numbers of long-period variables in the LMC. The *P–L* and *P–K* relations for carbon Miras derived here are based on the combined information contained in Feast et al. (1989, hereafter FGWC), Hughes (1989, hereafter H89), Hughes & Wood (1990, hereafter HW90) and Reid et al. (1995, hereafter RHG).

2 THE PERIOD–LUMINOSITY RELATION IN THE LMC

From HW90 42 spectroscopically confirmed carbon stars with well-determined periods and large amplitudes ($\Delta I \geq 0.9$, supposed to be Miras; their table 2) were selected. Two other spectroscopically confirmed carbon stars with well-determined periods (0520427 − 693637 and 0544397 − 740748) were not considered, as their published *JHK* colours are not consistent with their being carbon Miras.

From FGWC the 10 spectroscopically confirmed Miras were taken. One star (W103 in FGWC) is also listed in HW90. FGWC quote a period of 351 d, while HW90 quote 353 d; the period from FGWC is adopted for this star. The other nine stars from FGWC are not listed in H89 or in HW90, neither as definite nor as marginal LPVs.

RHG list six spectroscopically confirmed carbon stars with well-determined periods. None is listed by H89 or by HW90, but two are listed by FGWC. For 0515 − 6617

FGWC give a period of 211 d, while RHG give 223 and 225 d from two independent methods; a period of 224 d is adopted. For 0528 − 6520 FGWC give $P = 231$ d, while RHG give 230 d; the FGWC period is adopted.

The bolometric magnitudes ($m_{\rm bol}$) in FGWC were derived from mean *JHK* light curves using blackbody extrapolations to lower and higher frequencies; their infrared photometry is on the natural system of the SAAO 1.9-m telescope. The data in HW90 are single-epoch *JHK* observations on the AAO system, with bolometric magnitudes calculated from a colour-dependent bolometric correction applied to the *K* magnitude (Wood et al. 1983). RHG quote single-epoch *JHK* observations on the same system as FGWC. They calculate bolometric magnitudes using the same technique as HW90.

To construct a homogeneous data set, the following procedure was used. The AAO data (subscript *a*) from HW90 were transformed to the same system as the SAAO data (no subscript) using the following formulae:

$$\begin{aligned} (J-H) &= 0.965\,(J_a-H_a)+0.008, \\ (H-K) &= 0.968\,(H_a-K_a)-0.010, \\ K &= K_a-0.011\,(J_a-K_a)-0.008. \end{aligned} \tag{1}$$

Corrections for interstellar extinction were applied to the published photometry, assuming $A_J = 0.06$ and $A_H = 0.03$ and $A_K = 0.01$. The procedure followed by FGWC was then used to determine bolometric magnitudes. This involves using blackbody curves fitted to the measured fluxes in order to interpolate and extrapolate the data as a function of frequency. For the stars from FGWC the $m_{\rm bol}$ data were taken as published.

In Fig. 1 the *P–L* and *P–K* relations for the 56 objects are plotted. A least-squares fit to the data, excluding the clearly deviant points at $P = 657$ d (0454030 − 675031 in HW90) and 486 d (0524173 − 660913 in RHG), is $m_{\rm bol} = (-2.59 \pm 0.25) \log P + (20.52 \pm 0.63)$ with an rms deviation of $\sigma = 0.26$, and $m_K = (-3.56 \pm 0.17) \log P + (19.64 \pm 0.42)$ with an rms of $\sigma = 0.25$. The magnitudes from FGWC were derived from several observations (between five and 15) of each star. For this reason, when the above fits were made, the FGWC magnitudes were assigned a weight three times that of the single-epoch measurements taken from the other papers. The relations derived above are shown in Fig. 1 as solid lines, while the dashed lines are the relations derived by FGWC, namely $m_{\rm bol} = (-1.86 \pm 0.30) \log P + (18.73 \pm 0.74)$ with $\sigma = 0.13$, and $m_K = (-3.30 \pm 0.40) \log P + (18.98 \pm 0.98)$, with

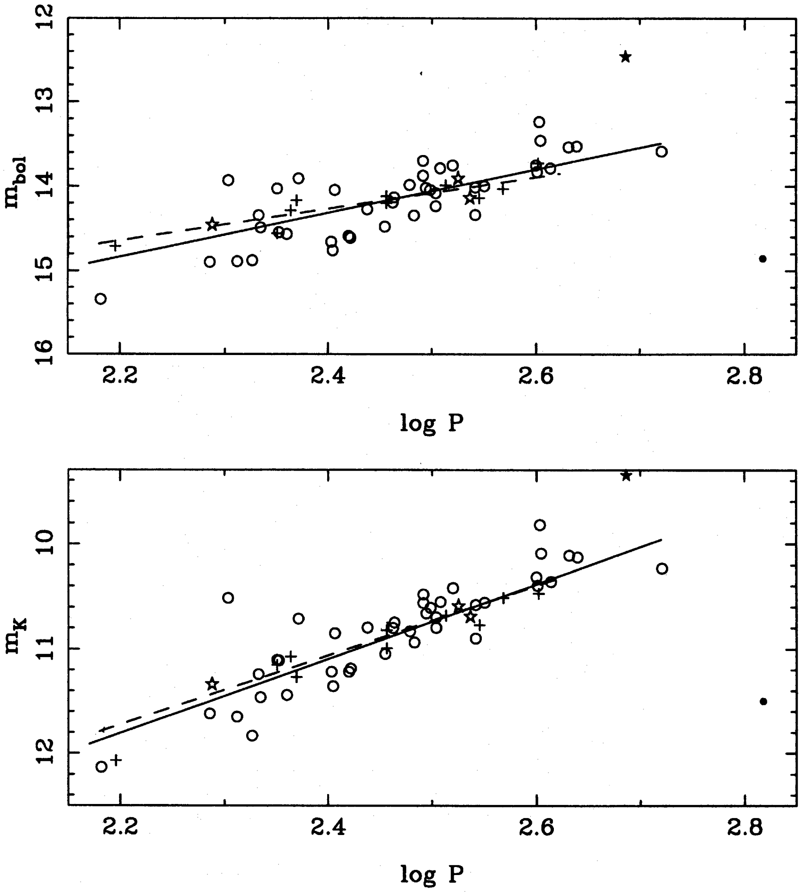

Figure 1. Top panel: the period–luminosity relation for spectroscopically confirmed carbon stars in the LMC. Points are labelled as follows: plus signs represent the 10 objects from FGWC; stars represent the four objects in RHG but not in FGWC, and circles represent the objects in HW90. The uncertainty in the individual $m_{\rm bol}$ values is smaller than the size of the symbols used. The solid line is a least-squares fit, excluding the deviant points at $P = 657$ d (0454030 − 675031 in HW90, filled circle) and 486 d (0524173 − 660913 from RHG, filled star): $m_{\rm bol} = -2.59 \log P + 20.52$. The dashed line is the relation by Feast et al. (1989). Bottom panel: as above, but for the *P–K* relation. The solid line is the best fit: $m_K = -3.56 \log P + 19.64$.

$\sigma = 0.18$. The new *P–L* relation is significantly steeper than that in FGWC, while the new *P–K* relation is very similar to the FGWC one.

The amplitudes, both of the *K* and of the bolometric light curves, observed by FGWC range from 0.4 to 1.2 mag (Glass et al. 1990). It seems reasonable to assume that the amplitudes of the stars selected from RHG and HW90 will also fall within this range. The mean bolometric magnitudes derived by FGWC are estimated to be within about 0.1 mag of the mean that would be derived from complete coverage of the light curves. The bulk of the scatter in the *P–L* relations derived above must therefore be a consequence of the spread introduced by single-epoch observations where mean values are preferred. Some contribution to the scatter will also result from the finite depth of the LMC, which has been estimated by Jacoby, Walker & Ciardullo (1990) to be 0.04 mag (1σ).

It is difficult to estimate the absolute accuracy of the bolometric magnitudes, and it is possible that the luminosity has been underestimated for any source with a circumstellar dust shell. Measurements over a larger wavelength range, of the type that should be possible from *ISO*, are required to establish properly the accuracy of these bolometric magnitudes. If there are systematic effects, they will probably be such that the luminosities of the longer period stars, which have thicker shells, will be underestimated compared to the shorter period stars, i.e., the true slope of the *P–L* relations may be slightly steeper than is estimated above. This does not affect the value of the Mira *P–L* relations for distance determination, provided that the bolometric magnitudes are determined by the method used here. Clearly, the technique, particularly the *P–K* relation, cannot be applied to stars with thick shells.

3 THE PERIOD-LUMINOSITY RELATION IN THE GALAXY

3.1 The distance modulus to the LMC and metallicity effects

In order to transform the relations from the LMC to the Galaxy, corrections for the distance, and possibly for metallicity effects, have to be made. A distance modulus to the LMC of 18.50 ± 0.13 is adopted (Panagia et al. 1991). The question of whether a metallicity term has to be applied is more controversial. Based on linear non-adiabatic pulsation calculations, Wood (1990) has derived relations linking the following stellar parameters: effective temperature, luminosity, mass and pulsation period. From these one may derive the following theoretical relation: $m_{\mathrm{bol}} \sim -1.6 \log P - 0.72 \log Z$ (neglecting possible variation in the mass of the star at a given period). With typical metallicities of $Z = 0.008$ and 0.02 in the LMC and the Galaxy, respectively, one therefore might expect Galactic LPVs to be about 0.29 mag fainter than LPVs in the LMC. A similar argument can be made for the relation between the *K* magnitude and the pulsation period, although the effect of metallicity is smaller than for the m_{bol}–P relation. Jura & Kleinmann (1992) applied a -0.25 mag shift in the zero-point of the *P–K* relation for oxygen-rich stars (corresponding to a -0.44 mag shift in m_{bol}; Wood 1990). Whitelock et al. (1994) showed that any metallicity effect on the zero-point is much smaller than that assumed by Jura & Kleinmann. Recently, Feast (1996) has shown that Wood's conclusion holds only if stars of the same mass but different metallicities become Miras at the same radius and thus at the same period. In light of these results, no shift in the zero-point was applied other than the distance modulus. The zero-point in the *P–L* relation for Galactic carbon Miras is therefore 2.02 (± 0.13) (the error due only to the uncertainty in the distance modulus).

3.2 The period and luminosity distribution of carbon stars in the Galaxy

The value of the zero-point in the *P–L* relation can be calculated directly if the pulsation period and luminosity are known for individual objects or in a statistical sense for a group of objects, assuming that the slope $\mathrm{d}M_{\mathrm{bol}}/\mathrm{d}\log P$ is the same for the Galaxy and the LMC.

From theoretical synthetic AGB calculations Groenewegen, van den Hoek & de Jong (1995) predicted that the carbon-star luminosity function in the Galaxy peaks at $M_{\mathrm{bol}} = -4.9$ (± 0.13). If the peak of the period distribution of Galactic Miras were known and it is assumed that there is no difference in the luminosity function of carbon Miras and of all AGB carbon stars, the zero-point of the *P–L* relation can be determined.

From a computer readable version of the General Catalog of Variable Stars (GCVS; Kholopov et al. 1985) all, i.e. 95, stars with an 'M' (for Mira) in the field of the type of variability and a 'C' (for carbon star) in the field of the spectral type were selected. Two of these were rejected; one, ST Sgr, because it is actually an S star, and the other, V1715 Cyg, because its Mira character is uncertain. 15 of the remaining 93 stars have no known periods, or poorly determined ones. For one, LP And, the period has been determined from its infrared light curves (Jones et al. 1990). The median period of the final 79 carbon Miras is 406 d. If all carbon stars with period determinations from infrared light curves (Jones et al. 1990; Le Bertre 1992) are added[1], the median shifts to 427 d (a sample of 102 stars).

A complication arises because Miras with longer periods are, on average, more luminous and therefore can be seen to larger distances in a magnitude-limited sample, as the GCVS may be assumed to be.

To correct for this effect, a *P–L* relation has to be adopted. Fortunately, the relative correction factors depend only on the slope $\mathrm{d}M_{\mathrm{bol}}/\mathrm{d}\log P$ (which is assumed to be the same for the Galaxy and the LMC) and not on the zero-point of the *P–L* relation. The relative, volume-corrected period distribution is calculated by dividing the number of stars in each period bin by $L^{1.5}$, where the luminosity is calculated from the *P–L* relation for the centre period of each bin. The median period of the volume-corrected period distribution is 366 d (including the Miras with periods from infrared observations). The exponent of 1.5 in the correction factor assumes that the limiting volume depends on the cube of the distance. This is valid if the limiting distance is smaller than the typical scaleheight. If the limiting distance is larger than the scaleheight, the limit-

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[1] The amplitudes in the *K* band indicate that these stars can be classified as Miras.

ing volume scales like the distance squared. In that case, the median period of the volume corrected sample is 390 d. Adopting 380 ± 15 d as the mean period of the volume-corrected sample and assuming $M_{\rm bol} = -4.9 \pm 0.13$ as the mean luminosity, a zero-point of 1.76 ± 0.14 is derived. This value is consistent at a 2σ level with that derived in the previous subsection. The error represents the internal error only. There are likely to be systematic effects due to the inhomogeneity and incompleteness of the GCVS, in particular for the longer period Miras which often are optically faint.

3.3 Individual objects

Two carbon Miras (UV Aur and RZ Peg) are thought to have binary companions (of spectral type B9V and F9V, respectively) and therefore offer the possibility for an independent determination of the luminosity. The spectral energy distributions (SEDs) of the companions are constructed (*UBV* photometry from Olson & Richer 1975) and fitted with Kurucz models representative of a B9 and an F9 dwarf. The interstellar extinction was assumed to be $A_V = 0.62$ (UV Aur + B9V) and 0.0 (RZ Peg + B9V), based on the $E(B-V)$ quoted in Olson & Richer for the companions. Luminosities of 95^{+85}_{-41} and 1.8 ± 0.3 L$_\odot$ are assumed for the B9 and the F9 dwarfs, respectively (Schmidt-Kaler 1982). The error corresponds to an uncertainty of one spectral subclass in the determination of the spectral type. Distances of $1.0^{+0.4}_{-0.3}$ (UV Aur) and 0.41 ± 0.03 kpc (RZ Peg) are derived. The distance to UV Aur agrees very well with the 1.4 ± 0.2 kpc determined by Reimers & Groote (1983). The second step is to use a dust radiative transfer model (Groenewegen 1993) to fit the SEDs of the carbon stars themselves to determine accurately their luminosity. Values of $11\,700^{+10500}_{-5000}$ L$_\odot$ for UV Aur and 740 ± 120 L$_\odot$ for RZ Peg are found.

There appears to be some controversy over the meaning of the 395-d period of UV Aur. It is classified as a symbiotic star and its period is sometimes attributed to orbital motion (e.g. Kenyon 1986), possibly due to a third star in the system. This appears to be unlikely, however, for the following reason. Kepler's law states $a^2 = GMP^2/4\pi^2$, where a is the orbital separation, P the orbital period, and M the total mass of the system. The total mass is likely to be between 1.2 M$_\odot$ (a white dwarf of 0.6 M$_\odot$ and a low-mass evolved carbon star) and 6.0 M$_\odot$ (a 3.5 M$_\odot$ B9 dwarf and a massive unevolved carbon star). With $P = 395$ d, it follows that $a = (1.7\text{–}2.9) \times 10^{13}$ cm. With an effective temperature of the carbon star of 2500 K and a luminosity of 11 700 L$_\odot$, its radius is 4.0×10^{13} cm. This implies that the orbital period must be longer than 395 d. In fact, polarimetric data show a cyclic behaviour with a period of 14 yr which is probably the orbital period (Khudyakova 1985).

The 395-d period in UV Aur is thus assumed to be due to pulsation, and it follows that $(M_{\rm bol} + 2.59 \log P)$ is $1.26^{+0.60}_{-0.70}$.

The luminosity derived for RZ Peg is low and is incompatible with the star being on the AGB. An F9 dwarf should have an initial mass near 1.2 M$_\odot$. The carbon star companion should initially be more massive than this. For a 1.2-M$_\odot$ star Boothroyd & Sackmann (1988) derived a luminosity of 1500 L$_\odot$ at the first thermal pulse. A more massive star will have a higher luminosity at the first thermal pulse. The star is then still oxygen-rich. If such a star is to become a carbon star, it will do so at a higher luminosity.

This is not the place to discuss the evolutionary status of RZ Peg in detail; it is noted simply that (1) the system may not be a binary, (2) the system may be a binary, but then RZ Peg is unlikely to be a carbon star on the AGB, and (3) if the companion is an F9 *giant*, the luminosity of RZ Peg would be about 1.2×10^4 L$_\odot$ (this is not to suggest that there is independent evidence to question the dwarf classification). In any case, RZ Peg is not a suitable object to determine the zero-point of the *P–L* relation.

4 DISCUSSION

The period–luminosity relation for carbon Miras in the LMC is derived to be $m_{\rm bol} = -2.59 \log P + 20.52$ for periods in the range $150 \lesssim P \lesssim 520$ d. This relation is different from the only other one for carbon Miras in the literature, by FGWC. Their relation is based on 20 stars, of which 10 are spectroscopically confirmed, while the present relation is based on 54 spectroscopically confirmed carbon stars. The new relation for carbon stars is similar to that for oxygen-rich Miras (FGWC): $m_{\rm bol} = -3.00 \log P + 21.35$. The luminosities (assuming a distance modulus of 18.5 to the LMC) of carbon- and oxygen-rich Miras are 2160 and 2290 L$_\odot$ at 150 d, and 5970 and 7420 L$_\odot$ at 400 d, respectively. Given the scatter in the two relations, these differences are not significant.

The period–luminosity relation of carbon Miras in the Galaxy is derived. The slope $\mathrm{d}M_{\rm bol}/\mathrm{d} \log P$ is assumed to be the same for the Galaxy and the LMC Miras. For oxygen-rich Miras this assumption has proved to be valid (see Whitelock et al. 1994 for a discussion). The zero-point has been derived from three different methods, and the results (2.02 ± 0.13, 1.76 ± 0.14, $1.26^{+0.60}_{-0.70}$) are internally consistent. Since the result from UV Aur is very uncertain, and the determination that involves the peak of the observed period distribution of Galactic carbon stars probably carries an additional unknown systematic error, the most reliable determination is that of simply shifting the *P–L* relation of the carbon Miras in the LMC. The finally adopted *P–L* relation for carbon Miras in the Galaxy is

$$M_{\rm bol} = -2.59 \log P + 2.02, \tag{2}$$

and the finally adopted *P–K* relation is

$$M_K = -3.56 \log P + 1.14. \tag{3}$$

The 1σ uncertainty is estimated to be about 0.26 mag in both equations; this includes the scatter in the relations for the LMC and the uncertainty in the distance modulus. Thus distances may be determined to an accuracy of about 12 per cent (1σ).

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