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Benchmark problems for dust radiative transfer

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ABSTRACT
When verifying a sophisticated numerical code, it is a usual practice to compare the results with reliable solutions obtained by other means. This work provides such solutions for the wavelength dependent dust radiative transfer problem. We define a set of benchmark problems in spherical geometry and solve them by three radiative transfer codes which implement different numerical schemes. Results for the dust temperature and emerging spectra agree to better than 0.1%, and can be used as benchmark solutions for the verification of the dust radiative transfer codes.

Key words: infrared – dust – radiative transfer: benchmark solutions

1 INTRODUCTION

Dust is abundant in the universe and many astronomical objects are associated with it. Most notable examples are galaxies and stars, at almost all points in their evolution. Dust surrounding an embedded source scatters, absorbs, and reemits radiation originating from the source. This processing usually results in an overall shift of spectral energy distribution to long, infrared (IR) wavelengths⋆. Embedded sources may be entirely obscured by the dust at optical wavelengths, and the only available information is obtained at IR wavelengths, invisible from earth until recently.

Thanks to the recent developments in IR techniques and facilities, the spectral energy distribution for many objects is now available, and the amount and quality of data are steadily increasing. Interpretation of these data can offer insight to the nature of optically obscured objects, making their IR signature a powerful tool of analysis. However, because of the complexity of the radiative transfer problem such analysis must be aided by sophisticated computational tools.

The wavelength dependent dust radiative transfer problem can be solved only numerically and early attempts to obtain approximate analytical and semi-analytical solutions assumed gray opacity (e.g. Chandrasekhar 1934, Kozirev 1934). These early works were based on the Eddington approximation that the radiation field is isotropic. The first direct numerical solutions were obtained by Hummer & Rybicki (1971) who employed iterations over variable Eddington factor (the ratio of the second to the zeroth moment of the radiation intensity). However, they also assumed a gray opacity, an approximation too crude for detailed analysis of observations.

The first calculations of the wavelength dependent dust radiative transfer in spherical geometry were performed by Scoville & Kwan (1976), and Leung (1976). Both attempts included some unrealistic assumptions and the first full approximation-free solution was obtained by Rowan-Robinson (1980, hereafter RR). His method is based on the direct integration of the radiative transfer equation, also known as ray tracing. Yorke (1980) developed a method based on iterations over variable wavelength dependent Eddington factors, a non-gray extension of the method originally used by Hummer and Rybicki. Both approaches remained the most widely used methods in subsequent developments of new codes (e.g., Groenewegen 1993) whose number has been steadily increasing during last two decades. Gradually, the codes' capabilities have been extended to two-dimensional geometries (e.g., Efstathiou & Rowan-Robinson 1990, Collison & Fix 1991, Men'shchikov & Henning 1997), and even to the three-dimensional case (Steinacker & Henning 1996). Yet, an important shortcoming remains in this development.

When verifying a sophisticated numerical code, it is a usual practice to compare the results with an analytical solution, or with a reliable solution obtained by other means. There is no such solution available for the wavelength dependent continuum radiative transfer problem. Definition of such a radiative transfer problem involves several complicated functions as its input (e.g., the spectrum of the embedded source, wavelength dependent grain optical properties), making analytical solutions impossible. The only practical

⋆ This is strictly true only in spherical geometry. Sources with toroidal, or disk-like dust distribution can show significant amounts of short wavelength radiation if viewed face-on.

approach is to compare solutions obtained by several independent codes for a set of well defined problems. If all such codes produce the same results, then it is likely that these results are reliable, and can be used as solutions of the benchmark problems for code verification.

This work defines a set of such benchmark problems in spherical geometry and compares solutions obtained by three radiative transfer codes which implement different numerical schemes. The benchmark problems are defined in Section 2, where we also briefly describe the radiative transfer codes which we used. Solutions for the dust temperature and emerging spectra are presented in Section 3, as well as the discussion of our results.

2 BENCHMARK PROBLEMS

Most of works describing infrared emission from astronomical sources present various computational results. Yet, it is difficult to use those results for the code verification. While many of the model parameters are usually explicitly listed, very often it is not easy to recognize all the underlying assumptions, nor to reproduce the grain optical properties used in the calculations. To avoid these problems, all input properties for the benchmark problems presented here are defined analytically. Such approach will ease comparison of the results obtained by other codes with those presented here.

2.1 Scaling Properties of the Radiative Transfer Problem

Traditionally, detailed modeling of IR radiation involved numerous input quantities, necessitating a large number of calculations to obtain successful fits, and diluting the value of the resulting success. However, as pointed out by RR, the number of relevant parameters can be drastically reduced by employing the scaling properties of the radiative transfer problem. For example, the luminosity of the central source is irrelevant, a fact by and large ignored in modeling astronomical sources.

The importance of scaling was recently emphasized by Ivezić & Elitzur (1995) who demonstrated that for given dust type, the IR emission from late-type stars can be successfully described by a single parameter — the overall optical depth τ[†]. All other quantities (luminosity, mass, mass-loss rate, etc.) enter only indirectly through their effect in determining τ, and thus are irrelevant in the modeling of IR emission. It was subsequently recognized that this powerful scaling is a general property of radiative transfer and that it can be extended to arbitrary geometries (Ivezić & Elitzur 1997, hereafter IE97). IE97 point out that the spectral shape is the only relevant property of the heating radiation when the inner boundary of the dusty region is controlled by dust sublimation. Similarly, the absolute scales of densities and distances are irrelevant; the geometry enters only through angles, relative thicknesses and aspect ratios. The actual magnitudes of densities and distances enter only through one independent parameter, the overall optical depth. Dust properties enter only through dimensionless, normalized distributions that describe the spatial variation of density and the wavelength dependence of scattering and absorption efficiencies. We now proceed to define the benchmark problems in terms of these fully scaled quantities.

2.2 Definition of the Benchmark Problems

A central point source is embedded in a spherically symmetric dusty envelope with an inner cavity free of dust. The source radiates as a black body at a given temperature T_*. The dust is in radiative equilibrium with the local radiation field, and this uniquely determines the dust temperature, $T_{\rm d}$, at every point in the envelope. The scale of $T_{\rm d}$ is determined by T_1, the dust temperature at the inner boundary, r_1. All radial positions can be scaled by this radius defining a new, dimensionless variable $y = r/r_1$. The dimensionless outer radius of the envelope, $Y = r_2/r_1$, is a free parameter. The dust density variation with y is assumed to be a power law $\propto y^{-p}$. The actual dust density, envelope size and opacity scales are all combined into a value for overall optical depth specified at some fiducial wavelength. We choose 1 μm for this wavelength and denote the corresponding optical depth by τ_1.

Dust optical properties are specified as scale-free wavelength dependent absorption and scattering opacities normalized to unity at 1 μm, $q_{\rm abs} = \kappa_{\lambda,\rm abs}/\kappa_{1,\rm abs}$ and $q_{\rm sca} = \kappa_{\lambda,\rm sca}/\kappa_{1,\rm sca}$, respectively. For spherical amorphous grains with radius a, $q_{\rm abs}$ and $q_{\rm sca}$ are roughly constant for $\lambda < 2\pi a$, and fall off as λ^{-1} and λ^{-4} for $\lambda > 2\pi a$, respectively. To mimic this behavior[‡] with analytical functions, we choose

$$q_{\rm abs} = q_{\rm sca} = 1 \tag{1}$$

for $\lambda < 1$μm, and

$$q_{\rm abs} = \frac{1}{\lambda} \tag{2}$$

$$q_{\rm sca} = \frac{1}{\lambda^4} \tag{3}$$

for $\lambda > 1$μm. The arbitrary choice of 1 μm as transitional wavelength is motivated both by typical astronomical grain sizes, and by desire to have such a transition at short wavelengths where its effects are most easily discernible. The chosen forms correspond to the maximal theoretically possible efficiencies for spherical amorphous grains with radius 0.16 μm (Greenberg 1971).

The above listed quantities fully specify the benchmark problems. We perform calculations for T_*= 2500 K, T_1= 800 K and Y = 1000, and produce eight different models: for two different density distributions, $p = 2$ and $p = 0$, and four values of optical depth, τ_1 = 1, 10, 100, 1000. Selection of different density distributions and a range in optical depth minimizes the chance that eventual inconsistencies in different numerical schemes would not be noticed.

2.3 Code Description

The three codes used in this work have similar approach to the solution of the radiative transfer problem, and none of

† Assuming a steady-state radiatively driven wind.

‡ We assume isotropic scattering.

them introduces any approximations (e.g., closure relation in the moment methods, Auer 1984). Since the problem cannot be solved analytically, the solutions are obtained on discrete spatial and wavelength grids. Within arbitrary numerical accuracy which determines the grid sizes, the solutions can be considered exact.

The spatial grids include the radial position and impact parameter grids, which also directly determine the angular grid needed for the integrations over solid angle. Sizes of the spatial grids range from ∼10 to several hundred points, depending on the method and overall optical depth for a given model. The wavelength grid has typically ∼100 points. To solve the radiative transfer problem means to determine the radiation intensity at each of the grid points, that is, to determine the wavelength dependent intensity as a function of angle for every radial position.

The radiative transfer equation is an integro-differential equation; in other words, the intensity at any grid point depends on the intensities at all other grid points. Thus the problem cannot be solved by straightforward techniques, and various numerical methods rely on iterations of different types. These iterations can be performed either for the intensity, or for its moments (energy density, flux, pressure, etc.). The codes used in this work implement different numerical schemes to perform the iterations, and we proceed with their brief descriptions.

Code 1 This code was originally developed by Yorke (1980) and generalized by Men'shchikov & Henning (1997) and Szczerba et al. (1996, 1997). The original version solves differential moment equations obtained by analytically integrating the radiative transfer equation multiplied by the powers of the direction cosine, over the solid angle (e.g. Auer 1984). Such an approach always results in a number of differential equations of one less than the number of unknown quantites (the moments of the radiation intensity). The closure relation can be expressed in terms of the variable Eddington factor, and this code calculates it directly from its defining equation after every iteration. Later versions of the code were extended to improve explicit ray tracing, with iterations repeated until convergence in dust temperature and mean intensity is achieved.

Code 2 Groenewegen (1993) has developed a code which solves the radiative transfer equation in spherical geometry from first principles, assuming isotropic scattering. The intensity is evaluated explicitly on sufficiently fine grids to obtain desired accuracy, and iterations are repeated until the convergence is achieved. This code was developed to allow for an explicit mass-loss rate dependence on time and velocity law, rather than a power-law density distribution. The model was tested against the results obtained with the model of Rogers & Martin (1984, 1986), for silicate grains up to an optical depth at 9.5 μm of 50; differences were 1% at most. For the purpose of the present paper, models with $p = 2$ (constant expansion velocity and mass loss rate) and $\tau_1 = 1$, 10, 100 were calculated ($\tau_1 = 1000$ was not included for this code because of unsatisfactory convergence).

Code 3 The third code, DUSTY, was developed by Ivezić, Nenkova & Elitzur (1997) and is publicly available. It solves the integral equation for the energy density obtained by analytically integrating the radiative transfer equation. The subsequent numerical integration is transformed into multiplication with a matrix of weight factors determined purely by the geometry. The energy density at every point is then determined by matrix inversion, obviating the need to iterate over the energy density itself. That is, unlike other codes, DUSTY can directly solve the pure scattering problem. The intensity is not explicitly evaluated, and can be easily recovered from the source function by using the weight factor matrix.

Table 1. Dimensionless parameter Ψ for eight benchmark models

p[(a)]	$\tau_1 = 1$	$\tau_1 = 10$	$\tau_1 = 100$	$\tau_1 = 1000$
2	3.48	5.42	13.1	84.1
0	2.99	3.00	3.10	3.75

(a) Power for the power-law describing the dust density distribution

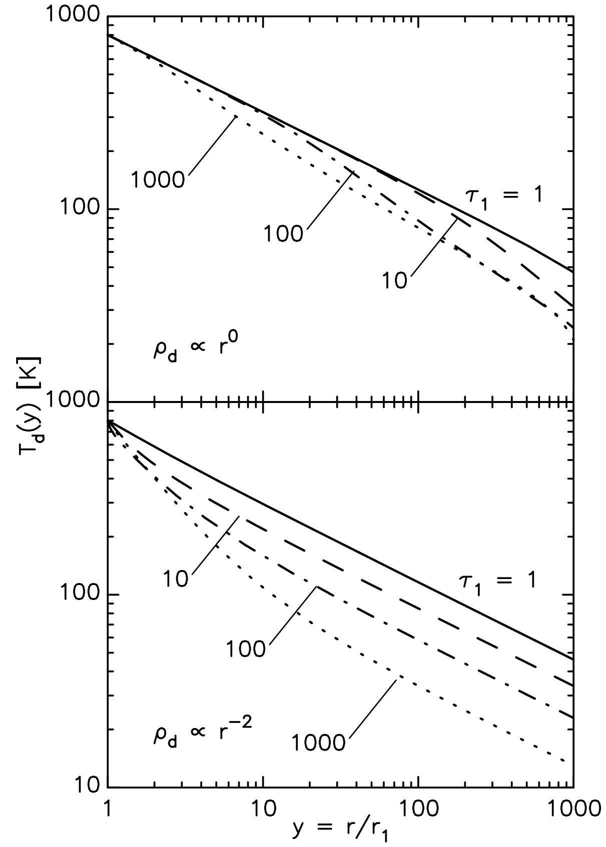

Figure 1. Dust temperature distribution through the envelope for two density distributions and optical depths at 1 µm as marked.

3 RESULTS

The full solution of the radiative transfer problem is contained in the radiation intensity. However, even in the spher-

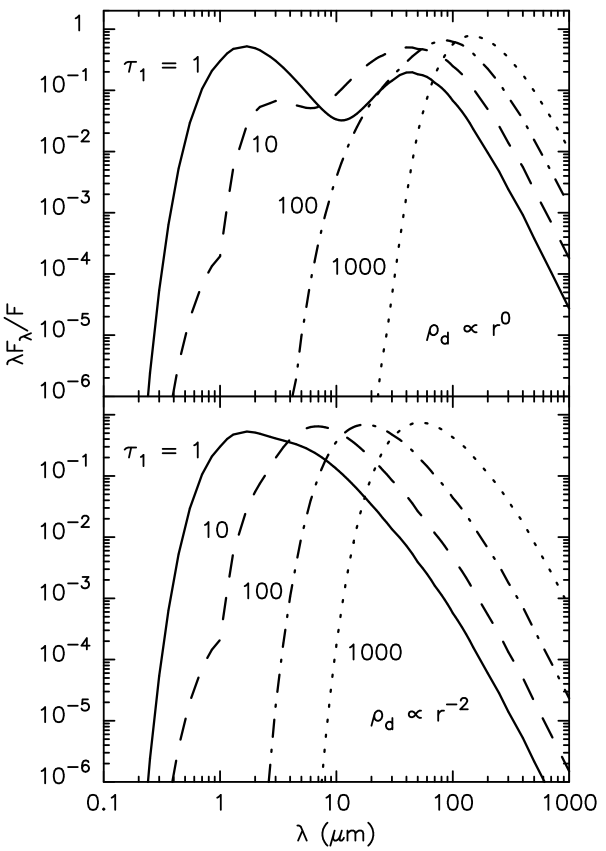

Figure 2. Spectral energy distribution of the emerging radiation for two density distributions and optical depths at 1 μm as marked.

ically symmetric systems it depends on three variables (position, angle, and wavelength) and its presentation would be quite involved. When isotropic scattering is assumed as here, the solution is also fully described by the energy density since it fully defines the source function. Equivalently, the emerging spectrum, the dust temperature distribution, and dimensionless parameter

$$\Psi = \frac{4\sigma {T_1}^4}{F_1}, \tag{4}$$

where σ is the Štefan-Boltzman constant and F_1 is the bolometric flux at y=1, also fully specify the solution[§] (IE97). That is, if these three quantities obtained by different codes agree, then the intensity distributions at every grid point agree, too.

The values for Ψ obtained for eight models calculated in this work are given in Table 1. All three codes agree within significant digits listed. Temperature distributions are presented in Figure 1. Again, the agreement is better that 0.1%. Emerging spectra are shown in Figure 2. They are presented as dimensionless, distance and luminosity independent spectral shape $\lambda F_\lambda / \int F_\lambda d\lambda$. Differences between the results obtained by different codes are smaller than the line thickness (< 0.1%).

The detailed numerical values for emerging spectra presented in Figures 1 and 2, and for all other relevant quantities can be obtained in computer readable form from Ž. Ivezić[¶]. These results can be used for the verification of the wavelength dependent radiative transfer codes, especially for the new multi-dimensional ones. Also, as the numerical radiative-hydrodynamical codes for modeling star and planet formation begin to include the wavelength dependent radiation transfer, the benchmark problems presented here might prove valuable for establishing confidence in the accuracy of their results (Boss 1996).

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REFERENCES

Auer, L.H., 1984, in Methods in Radiative Transfer, ed. W. Kalkofen (Cambridge: Cambridge University Press), p. 237
Boss, A., 1996, *private communication*
Chandrasekhar, S., 1934, MNRAS, 94, 444
Collison, A.J., Fix, J.D., 1991, ApJ, 368, 545
Efstathiou, A., Rowan-Robinson, M., 1990, MNRAS, 245, 275
Greenberg, J.M., 1971, A&A, 12, 240
Groenewegen, M.A.T., 1993, Chapter 5, Ph.D. thesis, University of Amsterdam, a copy of the relevant chapter can be obtained from the author, and a text-only version from http://www.mpa-garching.mpg.de/˜groen/groen.html
Hummer, D.G., Rybicki, G.B., 1971, MNRAS, 152, 1
Ivezić, Ž., Elitzur, M., 1995, ApJ, 445, 415
Ivezić, Ž., Elitzur, M., 1997, MNRAS, in press (IE97)
Ivezić, Ž., Nenkova, M., Elitzur, M., 1997, User Manual for DUSTY, Internal Report, University of Kentucky, accessible at http://www.pa.uky.edu/˜moshe/dusty
Kosirev, N.A., 1934, MNRAS, 94, 5
Leung, C.M., 1976, ApJ, 209, 75
Men'shchikov, A.B., Henning Th., 1997, A&A, 318, 879
Rogers, C., Martin, P.G., 1984, ApJ, 284, 327
Rogers, C., Martin, P.G., 1986, ApJ, 311, 800
Rowan-Robinson, M., 1980, ApJS, 44, 403 (RR)
Scoville, N., Kwan, J., 1976, ApJ, 206, 718
Steinacker, J., Henning, T., 1996, Proceedings of the ESO Workshop on "The Role of Dust in the Formation of Stars", 11-14 Sep. 1995, Garching, Germany, eds. Käufl, H.U. & Siebenmorgen, R., p. 355
Szczerba, R., Volk, K., Kwok S., 1996, Proceedings of the ESO Workshop on "The Role of Dust in the Formation of Stars", 11-14 Sep. 1995, Garching, Germany, eds. Käufl, H.U. & Siebenmorgen, R., p. 95
Szczerba, R., Omont, A., Volk, K., et al., 1997, A&A, 317, 859
Yorke, H.W., 1980, A&A, 86, 286

§ The absolute size of the inner boundary can be determined using Ψ, see eq. (27) of IE97

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