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Photoionization modelling of planetary nebulae – II. Galactic bulge nebulae, a comparison with literature results

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ABSTRACT

We have constructed photoionization models of five galactic bulge planetary nebulae using our automatic method, which enables a fully self-consistent determination of the physical parameters of a planetary nebula. The models are constrained using the spectrum, the *IRAS* and radio fluxes and the angular diameter of the nebula. We also conducted a literature search for physical parameters determined with classical methods for these nebulae. Comparison of the distance-independent physical parameters with published data shows that the stellar temperatures generally are in good agreement and can be considered reliable. The literature data for the electron temperature, electron density and also for the abundances show a large spread, indicating that the use of line diagnostics is not reliable and that the accuracy of these methods needs to be improved. Comparison of the various abundance determinations indicates that the uncertainty in the electron temperature is the main source of uncertainty in the abundance determination. The stellar magnitudes predicted by the photoionization models are in good agreement with observed values.

Key words: methods: data analysis – ISM: abundances – planetary nebulae: general.

1 INTRODUCTION

In Paper I of this series (van Hoof & Van de Steene 1999), we presented and tested a new method to derive simultaneously and self-consistently all physical parameters of a planetary nebula from a set of observed quantities. A modified version of the photoionization code CLOUDY (Ferland 1993) was used to calculate various models, searching for a best fit of the predictions to the observables in an automated way. This method uses emission line ratios, the angular diameter, and the radio and infrared flux to constrain the model. It also takes dust into account in the radiative transport. With this method we were able to determine the stellar temperature and luminosity, the inner, Strömgren and outer radius of the nebula, the density, the dust-to-gas mass ratio and the abundances. We investigated the accuracy of the determination of the physical parameters by applying this method to an artificial set of observables. First we proved that this method can pass a formal convergence test. Subsequently we introduced either measurement errors in the observables or changed the model assumptions, and investigated how this affected the best-fitting model. In this way we gained an understanding of the robustness of our method and hence of the reliability of the physical parameters. Our method was also compared with classical methods to determine the electron temperature and density and nebular abundances. It was shown that our method suffers less from noise in the spectrum than classical line diagnostics. However, this advantage may be lost if the model assumptions are not appropriate for the nebula being studied. The weakest points are currently the use of a blackbody approximation, the assumption that the inner dust radius coincides with the inner gas radius and the assumption of spherical symmetry.

Distance determinations of planetary nebulae (PNe) are still very problematic. Various methods are in use, but the range in distances obtained is often very large and no method has found general acceptance. Reviews of the current status can be found in Pottasch (1992), Terzian (1993) and Pottasch (1996). The lack of a reliable, generally applicable method to determine distances to PNe poses a problem when using photoionization models. To circumvent this problem we applied the method to a small sample of galactic bulge nebulae, which can be assumed to be all at the same well-known distance. Our aim is to study the accuracy of the determination of physical parameters by comparing our results with other literature values.

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A summary of the method and model assumptions is given in Section 2. The sample selection is presented in Section 3 and the modelling results in Section 4. Each PN in the sample is discussed individually, with special emphasis on the problems encountered during the modelling in Section 5. The resulting physical parameters are discussed by comparing them with results from other studies in the literature in Section 6. Our conclusions are given in Section 7.

2 SUMMARY OF THE MODEL ASSUMPTIONS AND THE METHOD

The model assumptions and the method were extensively described and discussed in Paper I. This section presents only a brief summary.

To model the planetary nebula, we use a modified version of the photoionization code CLOUDY 84.06 (Ferland 1993).

The model for the PN is quite simple, and comprises the following assumptions.

(1) The central star has a blackbody spectrum.
(2) The nebula is spherically symmetric.
(3) The density is constant inside the Strömgren radius of the nebula, and varies as $1/r^2$ outside.
(4) Dust grains are intermixed with the gas at a constant dust-to-gas mass ratio; if no information on the composition is available they are assumed to be a mixture of graphite and silicates.
(5) The filling factor, describing the small scale clumpiness of the gas, can be fixed at any value. If no information is available it is taken to be unity.
(6) The distance to the nebula is fixed by an independent individual or statistical method.

The above assumptions leave the following free parameters: the stellar temperature, the luminosity of the central star, the hydrogen density in the ionized region, the inner radius of the nebula, the dust-to-gas ratio, and the abundances in the nebula.

The outer radius of the nebula is not fixed as an input parameter, but calculated from the long wavelength end of the dust emission, or, as a fail-safe, when the electron density drops below $0.1\,\mathrm{cm}^{-3}$.

Adopting certain values for the input parameters, it is possible to calculate a model for the nebula with CLOUDY, predicting the continuum and line fluxes, photometric magnitudes (including the contribution of line emission) and the Strömgren radius.

To compare the model predictions with the observed quantities, a goodness-of-fit estimator is calculated. This estimator is minimized by varying all the input parameters of the model, using the algorithm AMOEBA (Press et al. 1986).

It is assumed that there exists a unique set of input parameters, for which the resulting model predictions give the best fit to a given set of observables. These input parameters are then considered the best estimate for the physical properties of the PN.

The full set of observed quantities necessary to derive the physical parameters of a PN are as follows.

(1) The emission line spectrum of the nebula. Usually this is an optical spectrum, but might also be an ultraviolet and/or infrared spectrum. The line ratios make it possible to constrain the stellar temperature, the density and the electron temperature in the nebula. They are also required to determine the abundances. For elements for which no lines are available, we assume standard abundances (Aller & Czyzak 1983).

(2) As dust is included in the model we also need information on the mid- and far-infrared continuum. For this the *IRAS* fluxes are used.

(3) To constrain the emission measure, either an optically thin radio continuum measurement (e.g. at 6 cm) is needed, or the absolute flux value of some hydrogen recombination line (usually Hβ).

(4) An accurate angular diameter Θ_d of the nebula is needed, which we define as $\Theta_\mathrm{d} = 2r_\mathrm{str}/D$. Here r_str stands for the Strömgren radius of the nebula and D is the distance to the nebula.

3 THE SAMPLE OF GALACTIC BULGE PNE

We selected a small sample of galactic bulge nebulae from Ratag et al. (1997, hereafter RPDM). Galactic bulge nebulae can be assumed to be all at a distance of approximately 7.8 kpc (Feast 1987). We chose the nebulae from RPDM because they published good quality spectra and also carried out their own photoionization analysis of the data, which we can use for comparison. The radio observations for these PNe are described in Gathier et al. (1983). The following selection criteria were used.

(1) The PNe should have a quality 2 or 3 *IRAS* 12-μm flux and quality 3 *IRAS* 25 μm and 60-μm fluxes.
(2) The absolute value for the radial velocity should be larger than $100\,\mathrm{km\,s}^{-1}$.
(3) The excitation class should not be labelled peculiar.

The resulting five PNe are presented in Table 1. All nebulae except M 2–4 are indicated by Acker et al. (1992) as likely bulge PNe. In view of the large radial velocity of M 2–4, $v_\mathrm{LSR} = -175.8\,\mathrm{km\,s}^{-1}$ (Gathier et al. 1983) it is unlikely to be a foreground object.

4 MODELLING RESULTS

In Table 2 we give the input values for the observables used for the modelling, together with the resulting model predictions. As can be seen from this table, not all the lines present in the spectra are predicted by CLOUDY, most notably the higher Balmer lines of hydrogen and several helium lines. Also the element chlorine is not included in the code. The resulting physical parameters for the nebulae are given in Table 3. The hydrogen density shown in this table is the constant density within the Strömgren sphere.

5 INDIVIDUAL REMARKS

The PNe in our sample all have nearly the same medium excitation class. This probably is partially a result of our selection

Table 1. Our sample of PNe selected from Ratag et al. (1997).

Name	PN G	α(2000) h	m	s	δ(2000) °	′	″
H 1–40	359.7 − 02.6	17	55	36.0	−30	33	33
M 1–20	006.1 + 08.3	17	28	57.5	−19	15	53
M 2–4	349.8 + 04.4	17	01	06.2	−34	49	39
M 2–23	002.2 − 02.7	18	01	42.6	−28	25	44
M 3–15	006.8 + 04.1	17	45	31.6	−20	58	02

Table 2. Comparison of the observed quantities (mostly taken from Ratag et al. 1997) and the model fit for our sample of PNe. The strength of the emission lines is given relative to Hβ = 100. The measured line fluxes have been dereddened. The entries [O II] λ 3727, λ 7325 and [S II] λ 4071 all stand for the entire multiplet. All observables for which entries in both columns 'obs.' and 'model' are present have been weighted in the goodness-of-fit estimator, except where indicated.

	λ	H 1–40		M 1–20		M 2–4		M 2–23		M 3–15	
Ion	Å	obs.	model	obs.	model	obs.	model	obs.	model	obs.	model
[O II]	3727	32.4[a]	38.5	55.5	83.1	97.1	125.2	14.2	20.4	48.6	101.5
H 12	3750							3.4			
H 11, O III	3771							4.2			
H 10	3798			5.5		4.0		5.1			
H 9	3835			6.7		7.1		6.5			
[Ne III]	3869	79.1	79.4	69.1	67.6	64.7	67.6	82.4	80.0	89.9	90.1
H 8, He I	3889	12.9		18.1		17.4		13.9		16.6	
[Ne III], Hϵ	3969	19.9		25.5		33.6		38.3		25.4	
He I	4026			2.7		2.7		1.82			
[S II]	4071	3.3	2.5		2.3	3.6	3.0	2.5	3.8		1.8
Hδ, N III	4102	25.8	29.4	25.6	30.3	24.7	30.5	24.0	30.2	26.1	29.5
C II	4267										
Hγ	4340	47.1	50.2	49.1	51.1	45.9	51.2	48.2	50.9	49.0	50.2
[O III]	4363	4.6[c]	12.0	7.5	5.4	2.9	2.1	13.9	9.9	3.3[a]	7.5
He I	4472	6.5	4.4	5.8	5.2		4.5	5.1	5.5	5.0	5.1
N III	4641									20.3	
He II	4686	17.1?[c]	1.9		0.4		0.14		0.6	4.1	3.5
[Ar IV], He I	4712			1.33		0.91		1.09		1.94	
[Ar IV]	4740	4.45	4.70	0.71	1.16	0.33[a]	0.69	0.80	1.42		6.67
Hβ	4861	100.	100.	100.	100.	100.	100.	100.	100.	100.	100.
He I	4922	1.36[a]				1.42		0.87			
[O III]	4959	307.	276.	336.[b]	315.	272.[b]	213.	304.	350.	328.[b]	219.
[O III]	5007	915.	827.	1009.[b]	946.	815.[b]	640.	1006.	1051.	983.[b]	658.
[N I]	5201		0.20		0.07		0.22		0.00		0.59
[Cl III]	5517							0.21			
[Cl III]	5538			0.47				0.25			
[N II]	5755	1.92[a]	2.16	0.92	1.05	1.39	1.52	1.19	1.06	1.0[a]	1.4
He I	5876	15.5	15.9	15.7	16.0	13.8	13.8	17.2	17.8	16.3	16.3
[O I]	6300	2.8	2.7	4.7	4.1	2.9	3.7	4.2	3.3	2.8	3.6
[S III]	6312	1.70	1.48	0.79[a]	1.12	1.59	1.33	2.4	2.3	1.24	1.00
[O I]	6364	0.88	0.88	1.46	1.35	1.00	1.23	1.42	1.09	0.79	1.19
[N II]	6548		19.9		14.6		32.1		6.4		18.8
Hα	6563	280.	278.	303.	269.	275.	268.	283.	269.	305.	278
[N II]	6584	61.4	59.7	45.4	43.8	85.6	96.2	18.9	19.2	57.6	56.3
He I	6678	3.8		4.0		3.2		4.7		4.6	
[S II]	6716	0.99	0.96	1.17	1.45	2.72	3.16	0.78	0.47	2.65	3.38
[S II]	6731	1.71	1.99	2.32	2.91	5.0	6.0	1.55	1.05	5.4	5.2
He I	7065	7.9	11.4	10.5	9.7	5.6	7.1	14.4	12.0	7.5	9.2
[Ar III]	7136	13.6	13.3	9.1	7.5	15.2	13.0	14.0	11.2	19.2	19.3
He I	7281			0.83		0.3[a]		0.94		0.59	
[O II]	7325	9.2	14.2	14.9	17.1	8.3	14.0	19.3	21.4	6.6	8.8
obs.	unit										
F_ν(12 μm)	Jy	2.38	2.38	1.13	1.00	0.56	0.53	1.93	2.10	< 0.53[d]	0.19
F_ν(25 μm)	Jy	18.45	19.11	3.94	4.44	5.00	5.83	9.31	6.54	5.66	6.02
F_ν(60 μm)	Jy	11.91	11.42	2.38	2.30	5.77	5.18	1.64	1.64	8.02	7.77
F_ν(100 μm)	Jy	< 73.48	3.22	< 4.59	0.66	< 12.59	1.73	< 126.70	0.24	< 10.39	2.72
F_ν(6 cm)	mJy	31.	31.0	47.	47.7	32.	32.2	41.	41.5	65.	65.4
Θ_d	arcsec	1.26	1.27	1.98	1.81	2.16	2.13	0.72	0.67	5.4	5.19
χ^2			0.63		1.68		2.28		4.17		2.75

[a] The value is uncertain.
[b] The sum of the intensities of the doublet was split using the ratio 3:1.
[c] This line was not weighted in the goodness-of-fit estimator χ^2, see also Section 5.
[d] This flux is not listed as an upper limit in the *IRAS* Point Source Catalog, see also Section 5.

criterion that the nebulae should have been detected by *IRAS* in the 12-μm band (criterion 1). Old bulge PNe, having a high excitation class and cool dust, might have insufficient 12-μm flux to be detected by *IRAS*.

In the rest of this section, each of the PNe in our sample will be discussed individually, with special emphasis on the problems encountered during the modelling.

5.1 H 1–40

Two lines were omitted from the list of observables for the following reasons. First, the He II λ 4686 line was omitted because the flux ratio given by RPDM is quite high, indicative of a high stellar temperature. However, the rest of the observational data are not consistent with such a high stellar temperature. Also, this line

Table 3. The physical parameters of the galactic bulge PNe in our sample determined with CLOUDY. Abundances of elements for which only one line was observed are marked uncertain. As we only model the core region of M 2–23, no values for the outer radius and total shell mass are entered.

	H 1–40	M 1–20	M 2–4	M 2–23	M 3–15
$\log(T_*/\mathrm{K})$	4.800	4.774	4.705	4.782	4.916
$\log(L_*/\mathrm{L}_\odot)$	3.798	3.607	3.555	3.639	3.663
$\log(n_\mathrm{H}/\mathrm{cm}^{-3})$	4.321	4.124	3.923	4.855	3.527
r_in/mpc	13	0.21	11	9	33
r_str/mpc	24	34	40	13	98
r_out/mpc	280[a]	360[a]	350[a]		520[a]
$M_\mathrm{ion}/\mathrm{M}_\odot$	0.042	0.092	0.088	0.015	0.47
$M_\mathrm{sh}/\mathrm{M}_\odot$	1.3[a]	2.3[a]	1.9[a]		6.5[a]
$\log\Gamma$	−1.70	−3.11	−2.50	−2.46	−2.60
ϵ(He)	10.96	11.02	10.96	11.05	11.03
ϵ(N)	7.78	7.81	8.13	7.67	7.60
ϵ(O)	8.23	8.72	8.84	8.67	8.22
ϵ(Ne)	7.39[a]	7.82[a]	8.15[a]	7.75[a]	7.53[a]
ϵ(S)	6.37	6.66	6.90	6.79	6.31
ϵ(Ar)	5.98	5.99	6.36	6.08	6.20[a]
T_e/kK	12.7	9.5	8.3	10.2	12.0
$\log U$	−1.40	+2.14	−1.16	−1.80	−1.58

[a] The value is uncertain.

is listed in table 3 of RPDM, but is not present in their table 1. Webster (1988, hereafter W88) took a spectrum of this PN but did not report the detection of this line. She should, however, have detected a line of the strength mentioned by RPDM. Tylenda et al. (1994) list an upper limit of five for the intensity of this line. In view of these uncertainties, we decided to omit this line. Because RPDM included this line in their modelling, this probably explains the higher stellar temperature they obtain.

The fitting of the [O III] λ 4363 line was also problematic. The observed flux was far too low to be consistent with the electron temperature predicted by our model. As the electron temperature derived from the [N II] line ratio is much higher (and more consistent with the value determined by our model), and also because the [O III] λ 4363 line is much stronger in the spectrum of W88 (however not as strong as predicted by our model), we decided that its value was too uncertain and omitted it from the input.

5.2 M 1–20

The intensity of the Hα line seems quite high, and is not fitted well. The discrepancy is too large to be attributed to measurement errors, hence this might indicate that the spectrum has not been sufficiently dereddened. There is however no evidence from the fits to the other lines to support this suspicion.

Our model gives a very small inner radius, also resulting in a very high ionization parameter. This is caused by the high *IRAS* 12 μm over 25 μm flux ratio, which might indicate the presence of hot dust (see also the discussion in Paper I).

5.3 M 2–4

The spectrum is fitted quite well, but there is a slight discrepancy for the [O III] $\lambda\lambda$ 4959 and 5007 lines. This is caused by the [O II] λ 3727 doublet, which is not fitted well. The latter doublet usually has a larger uncertainty owing to extinction and detector insensitivity.

5.4 M 2–23

This PN has the highest χ^2 of all PNe in our sample. This is mainly caused by the weak lines, which might indicate that this spectrum has a lower signal-to-noise ratio when compared with the other spectra. RPDM do not list error margins for their line flux ratios, so we had to assume reasonable values.

The model is not able to fit the *IRAS* 25 μm flux, which is very high compared with both the 12-μm and 60-μm fluxes. A possible explanation could be the presence of a 30 μm dust feature in the spectrum (Hoare 1990). This would imply that the nebula is carbon-rich, because this feature has only been observed in carbon-rich nebulae. The central star has spectral type Of (Aller & Keyes 1987, hereafter AK87).

The large difference between the optical diameter of 8.5″ (Acker et al. 1992) and the radio diameter of 0.72″ (Gathier et al. 1983) suggests that this nebula might be a core-halo nebula. All other evidence gathered in this paper is also consistent with this assumption and we will adopt it throughout the paper. As we used the radio diameter for the modelling, our model is only valid for the core region. The fact that our model is density bounded and gives a low ionized mass is consistent with the fact that we are only modelling the core region.

5.5 M 3–15

There is a suggestion of a systematic trend when comparing the observed and the modelled line flux as a function of wavelength. Also, the observed intensity of the Hα line seems quite high. This might indicate that the spectrum has not been sufficiently dereddened.

This PN has a [WC]-type central star (AK87). The central star temperature, the excitation class 5.5 (taken from RPDM) and the low *IRAS* 12 μm to 25 μm flux ratio all are consistent with an early spectral type: [WC3–4] (cf. Kaler 1989, Méndez & Niemela 1982, and Zijlstra et al. 1994, respectively).

The *IRAS* 12-μm flux is not listed as an upper limit in the Point Source Catalogue. However, when we used this value for the modelling, the resulting model was unrealistic. We therefore assume that the 12-μm flux suffers from confusion and took the quoted value to be an upper limit (see also the discussion in Paper I).

6 DISCUSSION

In this section, the modelling results are discussed by comparing them with results from other studies in the literature. Because distance-dependent parameters are usually not given by other authors, we will restrict ourselves to the distance-independent parameters of PNe.

6.1 Stellar temperatures

In Table 4 we present a comparison with the stellar temperatures given in the literature. These temperatures were derived using the Zanstra method and photoionization modelling. Results using the energy balance or Stoy method are not listed, because we consider this method, or at least the data for the nebulae being studied here, to be unreliable (Pottasch, private communication). One can see that the derived values agree quite well with only a few outliers.

Table 4. Comparison of the stellar temperatures for the PNe in our sample. The temperatures are given in kilokelvin. The abbreviations for the methods have the following meaning: H I – hydrogen Zanstra method, He II – helium Zanstra method, AM – photoionization modelling using model atmospheres, BB – photoionization modelling using the blackbody approximation.

H 1–40	M 1–20	M 2–4	M 2–23	M 3–15	Reference	Method
		55.	64.		1	H I
	53.		56.		2	H I
		51.5	65.0		3	H I
	65.		85.		4	H I
	49.5	49.9			5	H I
	59.9				5	He II
			50.	62.5	6	AM
80.0?	50.0	50.0	57.5	72.5	7	AM
64.					8	BB
63.1	59.4	50.7	60.5	82.4	9	BB

References: (1) Gleizes, Acker & Stenholm (1989) (2) Kaler & Jacoby (1991) (3) Pottasch & Acker (1989) (4) Tylenda et al. (1991a) (5) Mal'kov (1997) (6) Aller & Keyes (1987) using model atmospheres by Husfeld et al. (1984) (7) Ratag et al. (1997) using model atmospheres by Clegg & Middlemass (1987) and Husfeld et al. (1984) (8) Dopita et al. (1990) (9) This work.

Table 5. Various determinations of the electron temperature for the nebulae in our sample. The temperatures are given in kilokelvin. The abbreviations for the methods have the following meaning: ave. – average of [N II] and [O III]; model – average model prediction (see text).

H 1–40	M 1–20	M 2–4	M 2–23	M 3–15	Reference	Method
			18.0		1	[N II]
	17.1				2	[N II]
	12.5	12.5			3	[N II]
		10.2			4	[N II]
	10.4				5	[N II]
13.1	10.2	9.7	19.2	9.4[a]	6	[N II]
10.1					7	[N II]
			11.0		1	[O III]
	10.8				2	[O III]
	13.1	11.6			3	[O III]
		8.7			8	[O III]
			12.9		9	[O III]
	9.9				5	[O III]
9.3	10.4	8.5	13.0	8.4[a]	6	[O III]
9.7					7	[O III]
			12.6	11.2[a]	10	ave.
		11.1			11	ave.
12.7	9.5	8.3	10.2	12.0	12	model

References: (1) Acker et al. (1989) (2) Acker et al. (1991) (3) Costa et al. (1996) (4) Kaler et al. (1993) (5) Kaler et al. (1996) (6) Ratag et al. (1997) (7) Webster (1988) (8) Cuisinier, Acker & Köppen (1996) (9) Kaler (1979) (10) Aller & Keyes (1987) (11) Tylenda et al. (1991b) (12) This work.
[a] The value is uncertain.

The temperatures determined by our method agree well with the hydrogen Zanstra temperatures, with the single exception of the temperature for M 2–23 given by Tylenda et al. (1991a). As the other three determinations using the Zanstra method agree quite well, we assume the value given by Tylenda et al. (1991a) to be erroneous.

To derive stellar temperatures for photoionization modelling, sometimes certain line-ratios are used as temperature indicators (e.g. He II λ 4686 over Hβ). Especially for cooler central stars, where few temperature sensitive lines are available, this makes the determination dependent on one or two lines. Nevertheless, the results from the photoionization models are usually in good agreement. Exceptions are the temperature for H 1–40 derived by RPDM, and the temperatures for M 3–15. The deviating value for H 1–40 given by RPDM can probably be attributed to the He II λ 4686 line, which they used as a temperature indicator. We refer to the discussion in Section 5.1. For M 3–15, we find a higher stellar temperature than other authors. The largest discrepancy is with the value from AK87. This can probably be attributed to the fact that AK87 did not report a detection of the He II λ 4686 line in their spectrum (although a detection of roughly the same strength as in RPDM was reported in Aller & Keyes 1985). Because M 3–15 has a [WC]-type central star, part of the He II λ 4686 flux may originate from the central star. Unfortunately, no detection of the [Ar IV] λ 4740 line has ever been reported, so that no alternative temperature sensitive line is available. In view of this, the central star temperature for M 3–15 should be viewed with some caution.

We conclude that the temperature determination for the central stars in this sample is fairly reliable, although the situation for M 3–15 is not completely clear. This confirms our results from Paper I, in which we found the temperature determination to be robust.

6.2 Electron temperatures

In Table 5 the electron temperatures derived by different authors are compared. The electron temperature determined by CLOUDY is a weighted mean of the temperature in the nebula:

$$\overline{T_\mathrm{e}} = \frac{\int n_\mathrm{e}^2 T_\mathrm{e}\,\mathrm{d}V}{\int n_\mathrm{e}^2\,\mathrm{d}V}.$$

The observational material shows a large spread in most cases, even when the same method is used. This indicates that the electron temperature determination, at least in those cases where diagnostic lines have been used, is not very reliable. This is in agreement with our results in Paper I. Note the large difference between the [N II] and [O III] temperatures in the case of M 2–23. This difference is not caused by measurement error. For this particular object, the temperature derived from the [N II] lines has no physical meaning (Liu, private communication).

The electron temperatures derived from our method are in most cases just outside the range of values found with line diagnostics; three times at the low end and twice at the high end. The results from Paper I indicate that the electron temperature determination with our method should be robust. It is not apparent to us why the average values of the electron temperature derived from line diagnostics do not coincide with our results. This might indicate a problem, although the fact that we find both higher and lower results is not indicative of a systematic effect. Nevertheless, this issue should be investigated further in future research, using a larger sample.

6.3 Electron densities

In Table 6 the electron densities derived by different authors are compared. The electron density determined by CLOUDY is a weighted mean of the density in the nebula:

$$\overline{n_\mathrm{e}} = \frac{\int n_\mathrm{e}^3\,\mathrm{d}V}{\int n_\mathrm{e}^2\,\mathrm{d}V}.$$

There are enormous differences between the various determinations in the literature, even when the same method has been

applied. This indicates that the determination of densities with line diagnostics is unreliable, which confirms our results in Paper I. Also note the enormous differences between the [S II], [Cl III] and [Ar IV] densities for M 1–20 derived by Kaler et al. (1996).

Our values differ substantially from the values given by RPDM, although they are based on the same observational data. This is because we use a completely different method to determine the density. For three out of five nebulae we find results which are within the range of values found with other methods. For M 2–4 we find a value which is a bit larger. The results in Paper I indicate that our determination of the density is somewhat susceptible to measurement errors and errors in the model assumptions. This might provide an explanation for the discrepancy. The fact that we model only the core region of M 2–23 provides an explanation for the very high density we find for this nebula. Webster (1976) and Boffi & Stanghellini (1994, using the same spectrum) also find a high value using the [Ar IV] line ratio. The [Ar IV] lines are expected to be formed predominantly in the core region and hence this would confirm our results. On the other hand, the excitation in the core is too high to form large amounts of S^+. Hence the [S II] lines can be expected to originate predominantly from the halo and should therefore indicate lower densities. This of course also depends on the exact position of the slit over the nebula. All of this might be an explanation for the extremely large spread of values found for this nebula.

Table 6. Various determinations of the electron density for the nebulae in our sample. The densities are given in $10^3\,\mathrm{cm}^{-3}$. The abbreviations for the methods have the following meaning: radio – density determined from the radio flux, model – average model prediction (see text).

H 1–40	M 1–20	M 2–4	M 2–23	M 3–15	Reference	Method
			13.6		1	[S II]
10.9	4.7			24.3	2	[S II]
			3.0	2.5	3	[S II]
	17.8	4.5			4	[S II]
		7.0			5	[S II]
15.0					6	[S II]
		4.2			7	[S II]
	85.				8	[S II]
4.4	9.2	5.6	11.5	10.6	9	[S II]
				4.2	10	[S II]
		3.6			11	[S II]
35.1					14	[S II]
		5.7			7	[Cl III]
	7.8				8	[Cl III]
			79.		13	[Ar IV]
	1.0				8	[Ar IV]
			63.		12	[Ar IV]
13.5					6	radio
			20.		15	radio
	10.				16	radio
22.7	14.6	9.1	79.3	3.7	17	model

References: (1) Acker et al. (1989) (2) Acker et al. (1991) (3) Aller & Keyes (1987) (4) Costa et al. (1996) (5) Cuisinier et al. (1996) (6) Dopita et al. (1990) (7) Kaler et al. (1993) (8) Kaler et al. (1996) (9) Ratag et al. (1997) (10) Stanghellini & Kaler (1989) (11) Tylenda et al. (1991b) (12) Webster (1976) (13) Boffi & Stanghellini (1994) (14) Webster (1988) (15)

The quality of the data in Table 6 makes a comparison with our results meaningless. However, the data are at least consistent with the assumption that our results are more accurate than the results from line diagnostics.

6.4 Nebular abundances

In Table 7 we give a comparison of the abundances we determined with other literature values. We did not include the nitrogen abundance for M 3–15 from Henry (1990). After a discussion with Dr. Henry it was established that this abundance was flawed by an error in the analysis (as is also the case for the nitrogen abundances of M 4–3 and H 1–23 listed in the same paper; all other results are not affected). We also did not include the abundances for M 2–23 listed in Köppen, Acker & Stenholm (1991). It was established that this analysis was flawed by an error as well, and Dr. Köppen kindly provided us with a re-analysis of his data. The higher nitrogen abundance given by Walton et al. (1993) for M 2–23 might be a result of the inclusion of *IUE* data in their analysis. They systematically find higher nitrogen abundances for bulge PNe than other authors.

One can see that large differences can be found between the

Table 7. Comparison of the abundance determinations of the PNe in our sample.

	H 1–40					M 1–20					M 2–4			
Reference	1	2	3	4		5	6	2	4		5	7	2	4
ϵ(He)	11.03	11.06	11.04	10.96		11.07	10.94	11.02	11.02		11.11	10.96[a]	10.99	10.96
ϵ(N)	7.72	8.08		7.78		7.39	7.80	7.75	7.81		7.65	8.17	8.09	8.13
ϵ(O)	8.52	8.70	8.53	8.23		8.30	8.65	8.62	8.72		8.30	8.77	8.80	8.84
ϵ(Ne)	7.89	7.69		7.39[a]				7.79	7.82[a]				7.90	8.15[a]
ϵ(S)	6.88	6.77		6.37		6.43[a]		6.52	6.66		6.64	7.03	6.96	6.90
ϵ(Ar)	6.6	6.43		5.98				6.05	5.99		6.31	6.36	6.25	6.36

	M 2–23								M 3–15			
Reference	8	9	10	11	2	12	13	4	8	9	2	4
ϵ(He)	11.00	10.93		10.88[a]	10.98	10.92	10.96	11.05	11.03	11.01	11.03	11.03
ϵ(N)	7.68	8.20	7.70	7.55	7.40	8.13		7.67	8.08		8.14	7.60
ϵ(O)	8.40	8.42	8.18	8.34	8.22	8.47	8.11	8.67	8.41	8.51	8.74	8.22
ϵ(Ne)	7.60	7.62		6.46[a]	7.15	7.65		7.75[a]	7.48	7.62	7.86	7.53[a]
ϵ(S)	6.6			6.29	6.30			6.79	6.7[a]		6.86	6.31
ϵ(Ar)	5.75			5.86	5.81			6.08	6.5		6.53	6.20[a]

References: (1) Dopita et al. (1990) (2) Ratag et al. (1997) (3) Webster (1988) (4) This work (5) Costa et al. (1996) (6) Kaler et al. (1996) (7) Cuisinier et al. (1996) (8) Aller & Keyes (1987) (9) Henry (1990) (10) Kaler (1980) (11) Köppen (private communication) (12) Walton, Barlow & Clegg (1993) (13) Webster (1976).

[a] The value is uncertain.

various abundance determinations in the literature. If we exclude our own results, we find the following statistics. For elements heavier than helium, we find a difference between the lowest and highest abundance determination larger than or equal to 0.3 dex in 12 out of 22 cases, and larger than or equal to 0.5 dex in 4 out of 22 cases. For the helium abundances we find a spread larger than 0.1 dex in 2 out of 5 cases. Especially the abundances for M 2–23 show a large spread and should be considered uncertain. From this we draw the conclusion that, at least for the sample studied here, abundance determinations can not be considered very accurate. Uncertainties exceeding 0.2 dex to 0.3 dex are not uncommon.

When one compares the abundances for the individual PNe with the values from RPDM, one can see that for the two objects where the electron temperature is in good agreement (M 1–20 and M 2–4), the abundances also agree very well. For the other objects the abundance determinations differ. We attribute this to the difference in the determination of the electron temperature. When we compare our abundance determinations with the other values found in the literature, we see that our results often are slightly outside the range of values found by other authors. This behaviour is well correlated: either all outliers are at the low end or at the high end. This behaviour is also well correlated with our electron temperature determination. When our electron temperature determination is at the low end, our abundances are at the high end, and the reverse is also the case (see also Section 6.2). This indicates that the main source of uncertainty in the abundance determination is the electron temperature. Hence the discussion given in Section 6.2 applies here as well.

6.5 Stellar broad-band fluxes

As CLOUDY calculates the attenuation of the stellar continuum separately from the transport of the diffuse nebular continuum, we are able to predict broad-band photometric fluxes for the central star as they would appear through the nebula. In this way we could calculate a prediction for the Johnson *B* and *V* magnitudes. However, observed stellar magnitudes will be reddened due to interstellar extinction as well, and we have to take this into account in our predictions. To calculate the total extinction towards the nebula, we averaged all the measurements we could find in the literature. Since the continuum fluxes predicted by CLOUDY already take the internal extinction into account, we only have to correct the stellar magnitudes for the external extinction. Hence we used the internal extinction from our model, and subtracted it from the total extinction. Then we used this value for the external extinction to predict the reddened *B* and *V* magnitudes of the central star. Where necessary, we applied the interstellar reddening law given by Pottasch (1984). A comparison of the calculated values with the literature values taken from Acker et al. (1992) is given in Table 8.

The predicted magnitudes are slightly fainter than observed, but still in remarkable good agreement, considering the fact that we use a blackbody approximation to determine these values. Given the fact that a blackbody of a given temperature has more ionizing photons than a realistic spectrum with the same effective temperature, one can expect that in the best-fit model the total luminosity will be underestimated to compensate for this effect. However, we find that this effect is only very modest and this can be understood from the fact that we include the dust emission in the modelling. Grains can be heated very efficiently by Balmer continuum photons, as well as by Lyman continuum photons. Therefore, the *IRAS* fluxes give a good constraint on the Balmer continuum flux. This counteracts the previously mentioned underestimation of the total luminosity and explains the remarkable accuracy of our stellar broad-band fluxes.

6.6 Distances

In our model assumptions we assume the distance to be a fixed number. However, our method can easily be changed in such a way that the distance would be a free parameter. When this is done, the best-fit model would also give an estimate for the distance. We have investigated the possibility to determine the distance this way (van Hoof & Van de Steene 1996). We found that, though possible in principle, the spread in the resulting distance determinations is large. The distance determination is vulnerable to various observational errors, but especially to the error in the determination of the angular diameter. Since the angular diameter is notoriously hard to measure, this sensitivity makes the results very uncertain. When we determined the distances to the bulge nebulae in our sample with this method, we found the spread in the values to be larger than what is obtained from a statistical method (Van de Steene & Zijlstra 1995).

Closer investigation reveals that this method of determining distances is in essence identical to the method described in Phillips & Pottasch (1984). They already concluded that this method is unreliable. The use of a wrong value for the distance not only influences the distance dependent parameters but also some distance independent parameters, as was already discussed in Paper I. We therefore do not recommend this method, and advise the use of separately determined distances.

7 CONCLUSIONS

We applied our method which enables a fully self-consistent determination of the physical parameters of a PN, using the

Table 8. For the PNe in our sample we give in column 2 and 3 the Johnson *B* and *V* magnitudes resp. predicted by our model, in column 4 the internal extinction in the Johnson *V* band derived from our model, in column 5 the average total extinction derived from the Balmer decrement and the radio flux, in columns 6 and 7 the predicted reddened values for the Johnson *B* and *V* magnitudes and in column 8 and 9 the measured magnitudes given in Acker et al. (1992).

Name	$B_{\rm mod}$ mag	$V_{\rm mod}$ mag	$A_V^{\rm int}$ mag	$A_V^{\rm tot}$ mag	$B_{\rm pred}$ mag	$V_{\rm pred}$ mag	B mag	V mag
H 1–40	15.70	15.68	1.11	5.05 ± 0.26	20.86 ± 0.34	19.62 ± 0.26		
M 1–20	14.62	14.90	0.06	2.61 ± 0.10	17.96 ± 0.13	17.45 ± 0.10	17.7	17.1
M 2–4	14.41	14.64	0.14	2.78 ± 0.15	17.87 ± 0.19	17.28 ± 0.15	17.6	17.0
M 2–23	14.63	14.90	0.08	1.81 ± 0.22	16.90 ± 0.28	16.63 ± 0.22	16.7	
M 3–15	15.52	15.80	0.10	4.69 ± 0.25	21.53 ± 0.33	20.39 ± 0.25		

spectrum, the *IRAS* and radio fluxes and the angular diameter of the nebula, to a sample of five galactic bulge PNe. Comparison of the distance independent physical parameters with published data shows that the stellar temperatures generally are in good agreement and can be considered reliable. The literature data for the electron temperature, electron density and also for the abundances show a large spread, indicating that the use of line diagnostics is not reliable. Comparison of the various abundance determinations indicates that the uncertainty in the electron temperature is the main source of uncertainty in the abundance determination. The large spread in the literature data makes a comparison with our results meaningless. The stellar magnitudes predicted by the photoionization models are in good agreement with observed values.

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